Quantum Processes and Computation

Assignment 9, Monday, May 7, 2018

Exercise teachers:
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Handing in your answers: There are two options:

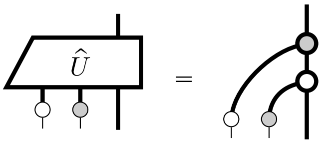
1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

Deadline: Friday, May 11, 17:00

Goals: After completing these exercises you can reason with (strong) complementarity and can do concrete calculations with ZX-diagrams. The total number of points is 100, distributed over 4 exercises.
Material covered in book: sections 9.2, 9.3, 9.4.

Note: In this exercise sheet ○ and ◍ will always represent strongly complementary spiders.

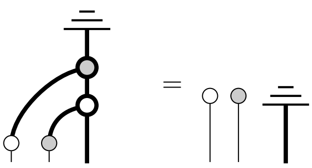
Exercise 1 (9.58) (30 points): In the lecture we discussed teleportation using complementary spiders, but to show that it is a valid protocol we must show that the actions of both Aleks and Bob can in fact be performed. We already know that Aleks's side is unitary, so it remains to show that Bob's side is as well, i.e. that



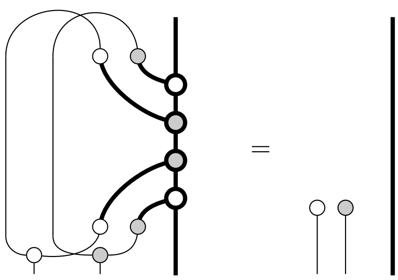
(1)

is a controlled unitary.

(i) Show that (1) is causal:



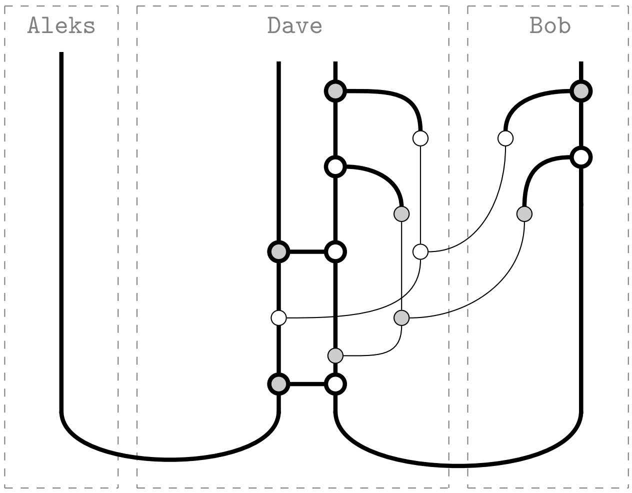
(ii) and that it is a controlled isometry up to a number (see section 8.4.2 or the previous exercise sheet for the definition):



(that it is a controlled unitary follows similarly).

In the future we might want to have a “quantum internet”[1] where users can do things using shared entangled state. However, the infrastructure needed for allowing any two users to directly create entangled states might be prohibitively expensive. The next exercise is about the protocol of *entanglement swapping*. In this protocol we have three parties, Aleks, Bob and Dave, where Aleks and Bob represent two users of our hypothetical quantum internet and Dave is a server. The protocol starts with the users Aleks and Bob sharing an entangled state with the server, Dave. We will show that Dave can perform some local quantum process and Bob some local correction so that in the end Aleks shares an entangled state with Bob. It is therefore sufficient for users to be able to create entanglement with a dedicated server, and perform local quantum operations in order to create a fully connected quantum internet.

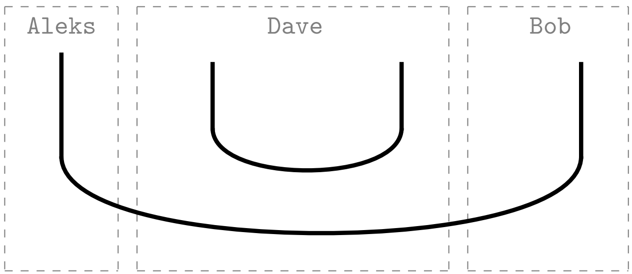
To be specific the protocol is:



(2)

So it consists of quantum CNOTs, measurements in ○ and ●, classical copies in ○ and ● and the controlled unitary (1).

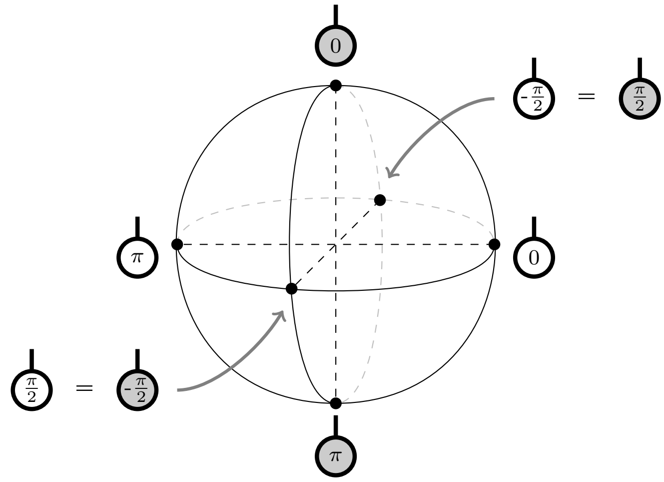
Exercise 2 (9.60) (30 points): Show that (2) is equal to:



Hint: this is very similar to the proof that teleportation works given in the lecture.

The ZX-calculus is based on the Z- and X-spiders and bases, but of course the Bloch sphere has a third axis: the Y-axis. The ‘Y-basis’ states can be represented in two different ways as Z- and X-phase spiders:

[1] That we will of course also want to connect to a centralised smart blockchain in the cloud.

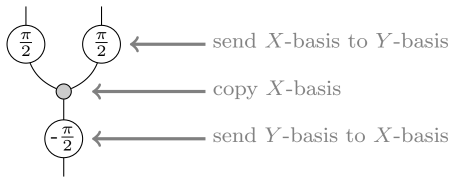


But in fact, these expressions are only equal because we have doubled the states.

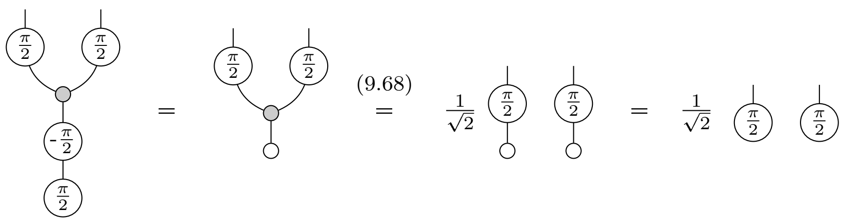
Exercise 3 (9.106) (15 points): Using the concrete definitions of the Z- and X-spider, show that

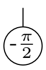

(3)

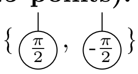
The two ways of writing the Y-basis states also allow us to find two different ways to copy these states. The first is:

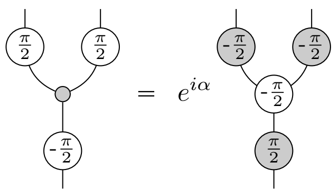


because



and similarly with  (see the text above Exercise 9.107 in the book).

Exercise 4 (9.107) (25 points): Using the equalities derived in the previous exercise, and by exploiting the fact that  forms a basis for $\mathbb{C}^2$, show that



for some fixed global phase $e^{i\alpha}$.

We will refer to this equality as the *Y-rule* in the future, and it will be important for us in the next lecture, as it allows us to change the colours of a spider in a diagram.