Quantum Processes and Computation

Assignment 2, Wednesday, February 6, 2019

Exercise teachers:
Aleks Kissinger (`aleks@cs.ru.nl`)
John van de Wetering (`wetering@cs.ru.nl`)

Handing in your answers: There are two options:

1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

Deadline: Tuesday, February 12, 12:00

Goals: After completing these exercises you know how to reason with cups, caps, and process-state duality in string diagrams. The total number of points is 70, distributed over 5 exercises. Material covered in book: everything up to section 4.2

Exercise 1 (3.38 & 3.40) (10 points): Suppose that there is a zero process $0 : A \to B$ for all possible types A and B (see Section 3.4.2).

(a) Show that for each type the zero process is unique. I.e. show that if $0' : A \to B$ is also a process satisfying the requirements of a zero process that then $0' = 0$.

(b) We call two processes f and g with the same inputs and outputs *equal up to a number* (written $f \approx g$) if there exist non-zero numbers λ, μ such that $\lambda f = \mu g$. Suppose a process theory has *no zero divisors*. That is, it satisfies the following property: $\lambda f = 0$ if and only if $\lambda = 0$ or $f = 0$. Show that $f \approx 0$ if and only if $f = 0$.

Exercise 2 (4.10 & 4.16) (20 points):

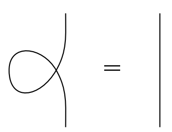
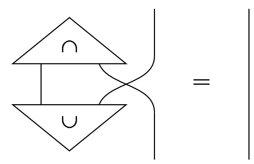
(a) Prove that in **relations**, the following relations on a set A:

 $:: * \mapsto \{(a,a) \mid a \in A\}$  $:: \forall a \in A : (a,a) \mapsto *$

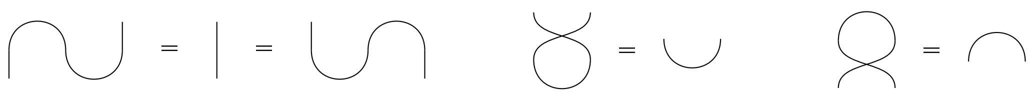
satisfy the *yanking equations* (eq. 4.11 in the book), and thus that **relations** has process-state duality.

(b) Show that process-state duality does not hold for **functions**.

Exercise 3 (4.12) (10 points): Prove that

 or written differently: 

follows from the following 3 equations:



Hint: Use the second notation (with the boxes) as it prevents you from accidentally cheating.

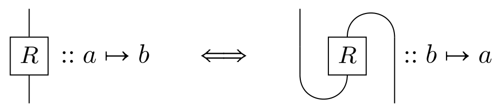
Exercise 4 (4.14 in PDF version) (20 points): Show that the following are equivalent:

(i) a state and an effect satisfying:

(ii) a state and an effect satisfying:

So in particular, if either eqs. (i) or eqs. (ii) hold, then all equations hold.

Exercise 5 (4.27) (10 points): Prove that in **relations** the following holds



i.e when a is related to b on the LHS, then b is related to a on the RHS (here the cup and cap are as in exercise 3). Note: In the next lecture we will see that the RHS is an instance of the *transpose* of a process.