Exercises on Description Logics and Frames

Exercise 1
Express the following sentences in terms of the description logic $\mathcal{ALC}$ (see slides):

a. All employees are humans.

b. A mother is a female who has a child.

c. A parent is a mother or a father.

d. A grandmother is a mother who has a child who is a parent.

e. Only humans have children that are humans.

Exercise 2
A knowledge base in description logics consists of a TBox and an ABox, i.e. $\text{KB} = (\text{TBox}, \text{ABox})$.

a. Construct a TBox describing a university. Use concept names such as $\text{University}$, $\text{Faculty}$, $\text{Education}$, $\text{Lecturer}$, $\text{Student}$, and role names such as $\text{teaches}$, $\text{works-for}$ and $\text{studies-at}$.

b. Extend the TBox from the previous item to a knowledge base by constructing an appropriate ABox.

Exercise 3
Consider the following knowledge base in description logic:

$\text{Female} \sqsubseteq \text{Human}$
$\text{Child} \sqsubseteq \text{Human}$
$\text{Works} \sqsubseteq \text{Human}$
$\text{StudiesAtUni} \sqsubseteq \text{Human}$
$\text{SuccessfulMan} \equiv \neg \text{Female} \sqcap \text{InBusiness} \sqcap \exists \text{married.} \text{Lawyer} \sqcap \exists \text{child.} \text{(StudiesAtUni} \sqcup \text{Works})$
$\text{Pedro} : \neg \text{Female}$
$\text{Pedro} : \text{InBusiness}$
$\text{Mary} : \text{Lawyer}$
$\text{John} : \text{Works}$
$\text{(Pedro, Mary)} : \text{married}$
$\text{(Pedro, John)} : \text{child}$

a. Translate this knowledge base to predicate logic.

b. Use resolution to derive that Pedro is a successful man.

Exercise 4
Which of the following statements are true? Explain your answer.

a. $\forall r.(A \sqcap B) \sqsubseteq \forall r.A \sqcap \forall r.B$

b. $\forall r.A \sqcap \forall r.B \sqsubseteq \forall r.(A \sqcap B)$

c. $\forall r.(A \sqcup B) \sqsubseteq \forall r.A \sqcup \forall r.B$

d. $\forall r.A \sqcup \forall r.B \sqsubseteq \forall r.(A \sqcup B)$

e. $\exists r.(A \sqcap B) \sqsubseteq \exists r.A \sqcap \exists r.B$

f. $\exists r.A \sqcap \exists r.B \sqsubseteq \exists r.(A \sqcap B)$

g. $\exists r.A \sqcup \exists r.B \sqsubseteq \exists r.(A \sqcup B)$

h. $\exists r.(A \sqcup B) \sqsubseteq \exists r.A \sqcup \exists r.B$
Exercise 5

a. Consider the following three frames frame taxonomy $T$:

```plaintext
class car is  
    superclass nil;
    wheels = 4;
    seats = 4 
end

class sportscar is  
    superclass car;
    seats = 2 
end

instance Rolls-Royce is  
    instance-of car;
    max-speed = enough 
end
```

Translate the information in the three frames to the description logics $\mathcal{ALC}$. Is the resulting set of formulae consistent or inconsistent? Explain your answer.

b. Consider the following algorithm for single inheritance with exceptions:

```plaintext:function Inherit(frame, attr-value-pairs)  
    if frame = nil then  
        return(attr-value-pairs)  
    end;  
    pairs ← AttributePart(frame);  
    attr-value-pairs ← attr-value-pairs ∪ NewAttributes(pairs, attr-value-pairs);  
    return(Inherit(Superframe(frame), attr-value-pairs))  
end
```

Dealing with exceptions is a form of non-monotonic reasoning. In the algorithm, the function $\text{NewAttributes}$ removes attribute-value pairs from $\text{pairs}$ that already occur in the parameter $\text{attr-value-pairs}$.

The algorithm operates on a tree-shaped taxonomy, but it is easy to extend it for graph-shaped taxonomies. Indicate how you can do this. Give an example showing that the extended algorithm is not always able to yield correct results for multiple inheritance with exceptions. Discuss ways to resolve these problems.

Exercise 6

a. Consider a frame formalism where attributes can have more than one value at the same time, i.e. all attributes are multi-valued. An attribute is in that case an expression of the following form: $a = \{c_1, \ldots, c_n\}$, $n \geq 1$, where $a$ corresponds to an attribute name and $c_1, \ldots, c_n$ are constants.

Now, consider the following frame:

```plaintext:class $F_1$ is  
    superclass $F_2$;
    $a = \{c_1, c_2, c_3\}$ 
end
```
where the keyword \texttt{class} indicates that \( F_1 \) is a class that has the (not shown) class \( F_2 \) as a superclass. Translate the frame to predicate logic.

Next, give an algorithm for inheritance in a tree-shaped taxonomy containing multi-valued attributes that is able to give correct results for inheritance.

b. Consider the following set of frames:

\begin{verbatim}
class polyhedron is
    superclass nil;
end
class prism is
    superclass polyhedron;
    base = (default 9);
    height = (value 2.9);
    volume = (demon if-needed V(base,height))
end
class cube is
    superclass prism;
    edge = (default 3);
    volume = (default 27);
    base = (demon if-needed O(edge));
    height = (demon if-needed H(edge))
end
instance cube1 is
    instance-of cube;
    edge = (value 2);
    volume = (default 9)
end
\end{verbatim}

Here we make use of so-called \textit{demons}, i.e. procedures that from the values of attributes that act as argument to the demon are able to compute values for the associated attributes. We make a distinction between ‘value’, ‘default’ and ‘demon’ facets of an attribute. These facets are processed in a particular order during inheritance. In this case, the demon \( V \) is defined as follows: \( V(x, y) = x \cdot y \), the demon \( O \) as \( O(x) = x^2 \), and the demon \( H \) as \( H(x) = x \).

Give a description of a particular form of inheritance that does implement a particular order in the processing of the facets such that the instance ‘cube1’ obtains the correct values for its attributes.

Give all the attributes with their values that are obtained by inheritance for the instance ‘cube1’. Explain how the results were obtained.