Abstract

Active automata learning is a technique used to learn models using membership/equivalence queries. Active learning consists of two phases, namely the exploration phase and testing phase. In the exploration phase, the learner constructs a hypothesis of the SUL (system under learning) by posing membership queries to the SUL. In the testing phase, the correctness of the constructed hypothesis is tested by posing equivalence queries and a counter-example is produced if a difference is observed, otherwise the learning is concluded. When learning models that exhibit far output distinction behavior, finding counter-examples between the constructed hypothesis and the SUL requires exploring many long test sequences, of which only one may refine the hypothesis further. In this thesis, we explore the impact that the inclusion of the most occurring subsequences in the test sequence may have in the learning of such models. We extract the most occurring subsequences from a generalized suffix tree built from the logs/traces of such models. We test our approach by running experiments on a family of Mealy machine that exhibit the far output distinction behavior. We conclude that our approach indeed helps with the learning of models that exhibit the far output distinction behavior.
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Chapter 1

Introduction

Specifying behaviors of large scale software components such as ASML’s TWINSCAN machine is complex due to these components often consisting of many other interacting components under the hood. However, representing these components using a Mealy machine and applying the concept of the active automata learning can help us derive behavioral specifications of these components.

Active automata learning can be traced back to Angulin’s seminal paper[2] on learning regular set through queries and counterexamples. Since then, the concept of active learning has been successfully applied in many fields across computer science. Common problems in computer science such as finding bugs in software[1], finding security vulnerabilities in network protocol[4] have been explored using active automata learning. The concept of active automata learning was also applied in evaluating whether legacy software and refactored implementation behave the same [6].

While active automata learning have been applied successfully in some of the aforementioned problems, there are also instances where active automata learning was not successful, for instance the learning of some of the industrial cyber-physical components in Aslam’s thesis [3]. Although the application of automata learning is quite effective, there are some components that were not learned using state-of-the-art learning and testing algorithms. A key problem, as pointed out by Aslam, is what is called the far output distinction behavior exhibited by some of the components. In this thesis, we are attempting to solve this very problem. The components as well as the hypothesis models are represented by Mealy machines. From this section onwards, we refer to components as models. The following sections will introduce the problem and guide you through the hypothesis of the proposed solution we have experimented with in this thesis.
1.1 Far output distinction behavior

Models where earlier choices affect the behavior much later on are said to show the far output distinction behavior. For completeness and ease of understanding, we point to Figure 1.1 provided in Aslam’s thesis, which displays a structural difference in the learned and reference model for one of the component that exhibit the far-output distinction behavior (FODB). Figure 1.1 (a) is the reference model and Figure 1.1 (b) is the learned model. As we can see from the figure, the path coloured green is missed by the algorithm, because a choice made in state 1 has an effect much later on. Hence, the algorithm fails to explore the path $a_1 b_2 b_4 b_6$, that distinguishes state 2 from state 6.

The active automata learning cycle consists of the following steps:

1. The model learning algorithm (learner) constructs a hypothesis model of the unknown component, referred to as system under learning (SUL).

2. Conformance testing techniques (CTT) are then used to construct test suites, which are used to check for equivalence between the hypothesis model and the SUL. If a counterexample is found, the hypothesis model is then refined and the learning cycle continues. Otherwise, the learning ends.

A test suite $T$ is usually constructed as a product of three sets namely an access sequence set $A$, permutation of input symbols $I^\leq n$ up to length $n$ and characterization set $C$ for a $n$ complete test suite, where $n$ is the number of extra states a hypothesis may have. A test suite $T$ is a $n$ complete test suite if the hypothesis has at most $n$ extra states with respect to SUL and the hypothesis passes $T$. A hypothesis passes $T$ if there is no difference in output observed between the hypothesis and SUL. A counterexample for
models with FODB consists of a long sequence of inputs capable of getting the system into states where different outputs can be seen. Finding these counterexamples has proven to be an expensive operation as, for a model with \( x \) input symbols, there are \( x^m \) combinations of sequences of length \( m \). In the worst case, of all the possible combinations, only one serves as a counter example that can further refine the hypothesis model [3]. The search space for these input sequences explodes exponentially with the increase of \( m \).

1.2 Our solution to the problem

The key problem lies in the use of conformance testing algorithms, which try randomly generated input sequences in efforts to find a counter-example. Our approach would be to extend this notion by including the subsequences (SS) of inputs that occur frequently from the logs of these models and use these subsequences as infixes during test generation. The test suite \( T \) then becomes a product of \( A, I^{\le n}, S \) and \( C \), where \( S \) is a set of the most occurring subsequences of at least length \( k \). A test \( t = w.x.y.z \in T \) (\( . \) here represents concatenation) will then have \( w \in A, x \in I^{\le n}, y \in S \& z \in C \). We can easily obtain these frequently occurring subsequences by:

- Constructing a generalized suffix tree (GST) from the inputs that occur in the logs/traces of these models.
- Extracting a set of frequently occurring subsequence from the generalized suffix tree.

Figure 1.2 displays the schematic view of our solution to the problem. The blue in the schema denotes the solution discussed above, whereas the black denotes the learning oracle. The research question to be answered in this thesis is as follows: How does the addition of frequently occurring subsequences in to the test sequences impact the learning of models that exhibit the far-output distinction behavior?

Chapter 3 and Chapter 4 will cover the algorithms used to generate a generalized suffix tree and extract subsequences from the GST. Chapter 5 will introduce and motivate the choice to conduct our experiments with the code-words family of Mealy machines. Succeeding this chapter comes Chapter 6 where we will discuss the setup of the experiments and discuss results of our experiment. Lastly, we end the document with Chapter 7 that discusses the conclusion from our experiments.

All the code related to generalized suffix tree can be found in the following repository https://github.com/Tomas2496/gsuffix. All the code regarding the experiments can be found in https://gitlab.science.ru.nl/bharat/tomas_bsc_thesis.
Figure 1.2: A schematic view of the solution
Chapter 2

Background

2.1 Mealy machines

We use the definition of Mealy machine from [7]. A Mealy machine $M$ is defined as a 6-tuple $\langle S, s_0, \Sigma, \Omega, \delta, \lambda \rangle$ where,

- $S$ is a finite nonempty set of states.
- $s_0 \in S$ is the initial state.
- $\Sigma$ is a finite input alphabet.
- $\Omega$ is a finite output alphabet.
- $\delta : S \times \Sigma \rightarrow S$ is the transition function.
- $\lambda : S \times \Sigma \rightarrow \Omega$ is the output function.

We extend $\delta$ to $\delta^* : S \times \Sigma^* \rightarrow S$ to handle sequences of input symbols. Similarly, we do the same with $\lambda$, hence $\lambda^* : S \times \Sigma^* \rightarrow \Omega^*$, where $\Omega^*$ is the set of sequence of output symbols.

2.2 Access sequence set

For a Mealy machine $M = \langle S, s_0, \Sigma, \Omega, \delta, \lambda \rangle$, an access sequence for a state $s \in S$ is a sequence of input symbols $i \in \Sigma^*$ such that $\delta^*(s_0, i) = s$. An access sequence set $A \subseteq \Sigma^*$ is a set that contains an access sequence for every state $s \in S$ in $M$.

2.3 Characterization set

For a Mealy machine $M = \langle S, s_0, \Sigma, \Omega, \delta, \lambda \rangle$, if $s, s' \in S$, $s \neq s'$ if $\lambda^*(s, i) \neq \lambda^*(s', i)$. A characterization set $C \subseteq \Sigma$ is a set of separating input sequence for every $s, s' \in S$ and $s \neq s'$.
2.4 Active automata learning

Active learning is a black box technique used to learn models using query-response mechanism [3]. The queries are then responded to appropriately and correctly by oracles that fulfill the minimally adequate teacher (MAT) model [2, 7]. Active automata learning comprises the learning algorithm (learner) and the model to be learned, commonly referred to as system under learning (SUL). There are two kinds of queries:

1. **Membership queries** are utilized to retrieve behavioral information on the SUL. The learner queries the oracle with input sequences and the oracle responds with the corresponding output sequences. These output sequences are collected and analyzed to construct a hypothesis of the SUL [7].

2. **Equivalence queries** are used to check for equivalence between the hypothesis and the SUL. A counter example is returned if there are diverging behaviors between the hypothesis and SUL. The counter example is further used to refine the hypothesis [7]. Otherwise, a boolean is returned to signify that the hypothesis and SUL are equivalent.

Figure 2.1 shows a schematic representation of the active learning cycle. The red edges represent the membership queries. The blue edges represent equivalence queries. The orange edges represent part of the equivalence queries the tester makes to check for equivalence. The tester has full access to the hypothesis model and can query input sequences to it. The tester queries the SUL with input sequence $i$ that is part of a test suite that is constructed for equivalence checking between the hypothesis and the SUL. The SUL responds with the output sequence $o$ for the queried input sequence $i$. The tester then compares $o$ with the output sequence retrieved when the hypothesis model is fed with $i$. Based on the comparison, the tester then responds to the learner with a counter example or with a boolean signifying that the hypothesis and SUL are equivalent.
Chapter 3

Suffix Tree

The chapter introduces the suffix tree data-structure and Ukkonen’s algorithm that is one of the most popular algorithms used to construct a suffix tree for a given word $\omega$. Furthermore, this chapter introduces terminologies, symbols and definitions that are vital in understanding the upcoming chapters.

3.1 Definitions

**Definition 2.1.1.** A tree is a directed graph $G = (V, E)$, such that $V$ contains a vertex $\gamma$, such that for each $v \in V$ there is a unique path from $\gamma$ to $v$.

**Definition 2.1.2.** A $k$-ary tree is a tree where all nodes have an upper bound on the number of child nodes, namely $k$. When the edge labels contain more than one symbol, the $k$-ary tree is called compressed, for instance, the edge labels of a suffix tree over word $\omega$ are subsequences of $\omega$.

A suffix tree is a compressed $k$-ary tree for word $\omega$ where $k = |\Sigma|$, where $\Sigma$ is the alphabet of word $\omega$. A suffix tree stores all the suffixes of a given string over the alphabet $\Sigma$. In this thesis, we are going to use an altered version of the definition provided by Ukkonen [8] for suffix-tree. In Ukkonen’s [8] definition, he introduces the notion of an auxiliary state $\bot$, that we do not utilize in this thesis for simplicity purposes. Throughout this document, we will use $STree(c_i)$ to denote suffix tree for some word $\omega$ until the character $c_i$.

**Definition 2.1.3.** A suffix-tree can be defined using a 5-tuple $\langle Q, \text{root}, F, g, f \rangle$, where

- $Q$ is a set of (explicit/branching) states of the suffix-tree.
- $\text{root}$ is a state representing the root of the tree.
- $F$ is a set of leaf states of the tree, $F \subset Q$. 
• $g$ is a transition function that takes a reference pair of a state as input and transitions to the appropriate state in the tree.

• $f$ is a function that of type $Q \rightarrow Q$ that gives the suffix link of the given state.

The numbers in the leaf state corresponds to the start index of the suffix, for instance in fig 3.2 the leaf state with index 2 represents the suffix $nse$ of word sense.

There are two types of states in a suffix tree:

• Branching states (Inner nodes and root) and leaf states are explicit states.

• Implicit states are present in the edge labels. Consider the edge with label $nse$ in fig:3.2. The edges in a suffix tree are compressed, therefore there are hidden states in edges. These states can be seen in fig:3.1.

![Figure 3.1: Implicit states of edge nse

We are going to follow Ukkonen’s way [8] of referencing states, as it goes very well with understanding the algorithm. We use the reference pair $(s, w)$ to refer to an explicit or implicit state r of a suffix tree, where $s$ denotes an explicit state that is an ancestor of r and w denotes the string that is written out by the transitions from s to r in the appropriate suffix tree. When s is r’s nearest ancestor (making w the shortest feasible), a reference pair is said to be canonical. The canonical reference pair for an explicit r is, of course, $(r, \epsilon)$. Once more, we encode the string w as a pair of pointers $(k, p)$ so that $t_k...t_p = \omega$. A reference pair $(s, w)$ acquires the form $(s, (k, p))$ in this manner. The symbol for the pair $(s, \epsilon)$ is $(s, (p, p))$.

A suffix tree for a set of strings over the alphabet $\Sigma$ is called a generalized suffix tree. The leaf state of a generalized suffix tree stores a pair, $(wordNumber, startIndex)$ as it can be seen in figure 3.3.

**Definition 2.2.3.** Let state $a$ and $b$ be two states such that the path to $a$ is $x\alpha$ and the path to state $b$ is $\alpha$, where $|x| = 1$ and $|\alpha| >= 0$ then there is an edge from state $a$ to $b$ and this edge is called suffix link. Suffix links are represented by dotted arrows, as can be seen in figure 3.2.
3.2 Ukkonen’s Algorithm

Ukkonen’s algorithm is an online algorithm that builds the suffix tree, traversing the string character by character from left to right. Let \( \omega = c_i c_{i+1} \ldots c_n \) be a string over some alphabet \( \Sigma \). The core idea behind Ukkonen’s algorithm is building \( STree(c_{i+1}) \) from \( STree(c_i) \) and so forth until the end of the string. The algorithm does this by traversing the boundary path.

**Definition 2.3.1.** A boundary path is the path that starts from the deepest state and ends up at the root following the suffix links. Boundary path can be seen in figure 3.4.

**Definition 2.3.2.** An open transition is the transition that leads to a leaf state \( s \in F \). In Ukkonen’s algorithm, while building suffix tree for string \( \omega \), every leaf state \( x \)’s reference pair is \((a, (y, \infty))\), where the end index is updated to \(|\omega| - 1\) when the tree is built.

**Definition 2.3.3.** An active point is the first state that is not reached by an open transition along the boundary path.

**Definition 2.3.4.** End point is the state that already have \( x \) transition when \( x \) is being inserted in the suffix tree.

To understand how Ukkonen’s algorithm works, let’s illustrate the working of the algorithm with an example, let \( W = c_1 c_2 \ldots c_{i-1} c_i \) be a word over the alphabet \( \Sigma \) and \( STree(c_{i-1}) \) be the tree we currently have. To extend \( STree(c_{i-1}) \), the algorithm traverses the boundary path by adding a \( c_i \) transition to each of the states it encounters along the path. However, there are two main groups of states to which \( c_i \) transitions must be added as Lemma 1 [8] states. The one group consists of states reached by open transitions and the other group consists of states starting from the active point until
• **Group 1 Open transitions states**: The transition leading to this state is an open transition and we just append the end index of the reference pair for this state.

• **Group 2 States from Active point till end point**:
  
  – **Case 2a**: If the next character on the edge is $c_i$ then adjust the suffix link for the next extension.
  
  – **Case 2b**: Make the current state explicit if it is not explicit by branching the current transition in two transitions. One transition leading to the old transition and another transition for new open $c_i$ transition. Following that, adjust the suffix link for the next extension.

**Definition 2.3.5.** An *explicit suffix* tree for a string $\omega$ over alphabet $\Sigma$ is a suffix tree that contains all the suffixes of $\omega$ explicitly. In Laymen’s term, there is a path from the root to a leaf for every suffix of $\omega$.

**Definition 2.3.6.** *Canonical reference pair*: Let $(r, (s, e))$ be the reference pair of the state $w$ then $(r, (s, e))$ is canonical if $r$ is the closest explicit ancestor of state $w$ and $s$ is the updated accordingly but $e$ remains the same.
Each string $\omega$ that must be added be terminated with a unique terminal character to ensure the construction of an explicit suffix tree. The reason that a unique terminal character say $\$ makes a tree explicit is because it enforces the addition of $\$ transition, thereby ensuring the explicit presence of each suffix of the string $\omega$. The above steps are repeated by making sure that the reference pair to the states are canonical before adding the next transitions. The complete Ukkonen’s algorithm in pseudocode can be seen down below. In our implementation, canonize and update are both combined in to a single procedure called run\_phase. The run time complexity as stated in [8] is $O(n)$, where is the length of the string $\omega$ for which the suffix tree is being built.

**Algorithm 1**  Ukkonen algorithm, where $\omega$ is string over some alphabet $\Sigma$ and $\$ is terminal character

1: **procedure** UKKONEN(root, $\omega\$$)
2:     **for** $i \in \omega$ **do** RUN\_PHASE($i$)
3:     **end for**
4:     **dfs to add leaf state indices**
5: **end procedure**
Algorithm 2 The procedure that extends the tree with $i$

1: procedure RUN_PHASE($i \in \Sigma \cup \{\$\}$)
2:     while there exists a suffix to be added do
3:         if active point is implicit state then
4:             if active point not canonical then
5:                 canonize active point
6:             continue
7:         end if
8:         if the next character on transition is $i$ then
9:             update active point
10:            break
11:         end if
12:         create new inner state with transition $i$.
13:         update active point
14:         update suffix link
15:     else
16:         if transition $i$ exists then
17:             update the active point
18:            break
19:         end if
20:         add transition $i$ to the active point
21:         update active point
22:         update suffix link
23:     end if
24: end while
25: end procedure
Chapter 4

Generalized suffix tree algorithm

This chapter describes the algorithm that we use to construct a generalized suffix tree. Moreover, this chapter also includes the algorithm that is used to extract the most occurring substring of minimum length \( n \) from a given generalized suffix tree.

4.1 Generalized suffix tree algorithm

This section explains the algorithm used to generate a generalized suffix tree. The algorithm used is an extension of Ukkonen’s algorithm for on-line construction of suffix tree [8], this algorithm was explained in Chapter 3.

Before we jump to our approach, our approach combines the two methods suggested in Gusfield’s book [5]. The first approach concatenates all the strings \( \omega \) with different terminal character \( t \) and run Ukkonen’s algorithm described in algorithm 1 in chapter 3. However, this tree contains suffixes that occur in-between two strings. The second approach, that is immune to this issue, is to add each string \( \omega \) terminated with a terminal character \( t \) one after the other to the tree by first running down the tree until a mismatch occurs and then running Ukkonen’s algorithm from the mismatched character. The issue with the second approach is that the labels in the transitions may correspond to different strings, thus the labels of transitions have to store the string that resulted in the creation of the transition. The approach we assume in this thesis is the combination of the two methods described in Gusfield’s book [5].

The basic idea behind the algorithm is to concatenate all \((\omega_0, \omega_1, \ldots, \omega_n)\) \( n \) strings using different unique terminal characters \((t_0, t_1, \ldots, t_n)\) for each string \( \omega_i \), \( 0 \leq i \leq n \) to get \( \Omega = \omega_0 t_0 \omega_1 t_1 \ldots \omega_n t_n \). Following this, we construct a suffix tree for the string \( \Omega \). However, this method requires the removal of the suffixes that occur between the strings, as a generalized suffix tree
should only contain suffixes of the strings $\omega_i, 0 \leq i \leq n$. Therefore, some transitions need to be pruned.

There are two questions that need to be answered in order for us to understand the need for the unique terminal characters and the removal of the in-between strings occurring suffix that we do not require our tree to store. These questions are as follows :

4.1.1 Are N different terminal characters for N strings required ?

The idea behind requiring a unique terminal character is to make the current suffix tree explicit. Let $\Omega = \omega_1 t_1 \omega_2 t_2$ be a concatenation of two strings with unique terminal characters $t_1$ and $t_2$ respectively. After string $\omega_1 t_1$ has been added to the tree we have all suffixes of $\omega_1$ in the tree explicitly and the root is the active point. We then proceed to add the second string $\omega_1 t_2$. We can take two things away from the above mini run-through of the algorithm and they are as follows :

- We need at-least one unique terminal character.
- We need to start at root for adding the next string.

These two above points do pose the following question. How does the tree know that a character $x$ is unique to it?

The root of a tree has $x$ transitions $x \geq 0$. All these transitions must be unique. If there is a transition $y \in \Sigma$ from the root of the tree then the character $y$ is not unique to the tree.

Since Ukkonen’s algorithm uses the boundary path to add the transitions, this implies that the root gets the new transitions at the end. Therefore, if we skip adding the unique terminal character $t$ transition to the root of the tree, $t$ always remains unique to the tree. Consequently, for $n$ different strings we do not require $n$ different unique terminal characters.

4.1.2 How to avoid adding the in-between strings occurring suffixes into tree ?

In Ukkonen’s algorithm, the end index of all leaf states $x$ are updated only after the tree is built. This works perfectly in Ukkonen’s algorithm because we are dealing with only one string $\omega$ which consequently have only a single end index which is $|\omega| - 1$. However, when dealing with multiple strings, we have multiple end indices in the leaf states of a generalized suffix tree. Moreover, the active point is not always the root at the start of adding the suffixes of the next string. This happens when we add a suffix to the tree that is already present in the tree. To account for these cases, we do the following :
• Since each string is separated by a terminal character $t$, we can update the end index of a leaf state when we reach the end of a string in the run of the algorithm.

• We reset the active point back to root for adding the next string.

After applying the following changes the pseudocode for the extension of Ukkonen’s algorithm for generalized suffix tree looks is given below. The run_phase$$(i \in \Sigma \cup \{$$}$$) procedure remains mostly the same with just an addition of the following code after line 15.

1: if active point is root and i is $$ then
2: break
3: end if

Algorithm 3 Ukkonen, where Ω is a string over some Σ \cup \{$$} and $$ is terminal character
1: procedure UKKONEN(root, Ω)
2: for i = 0, 1, ..., |Ω| - 1 do
3: if i > 1 and Ω[i - 1] = $$ then
4: update the end index of leaf states
5: reset active point
6: end if
7: run_phase(i)
8: end for
9: dfs to add leaf state indices
10: end procedure

4.1.3 Complexity analysis of algorithm 3
We divide the complexity analysis of Algorithm 3 in two following parts.

Lines 1 - 8
The input to the generalized suffix tree algorithm Ω and we construct the tree traversing each character from left to right using Ukkonen’s algorithm. Therefore, run-time complexity of lines 1-8 of the Algorithm 3 is $O(|\Omega|)$.

Line 9
We know that there are $n$ leaf states for a suffix tree for a string $\omega$ of length $n$. Since the number of internal states $i$ increases the number of leaf states $n$ by at least 1, we can say that $n - 1 \geq i$. We also know that there is only 1 incoming transition to every state except the root, so this means that the total number of transitions $t$ is equal to the sum of $i$ and number of leaves
Therefore, \( t = n + n - 1 = 2n - 1 \). There are \( 2n - 1 \) transitions and \( 2n - 1 \) states other than the root. The total number of transitions and states in a suffix tree built for a string \( \omega \) of length \( n \) is \( 2(2n - 1) + 1 = 4n - 1 \).

Similarly, a generalized suffix tree built for \( \Omega \) will have at most \( |\Omega| - n \) leaf states. Repeating the above calculation for a generalized suffix, we conclude that the total number of states (including root) and transitions for a generalized suffix tree built for \( \Omega \) is \( 4(|\Omega| - n) - 1 \).

All things considered Algorithm 3 in the worst case have a run time complexity of \( O(4(|\Omega| - n) - 1) \).

### 4.2 Most occurring substring of minimum length \( n \) algorithm

We are given a generalized suffix tree from which we must extract the most occurring substring of minimal length \( n \). Let’s break down the problem statement i.e. most occurring substring of minimum length \( n \) by first extracting the most occurring substring regardless of the length and then following that focusing on the length of the substring. As previously done throughout this thesis, we first analyze the problem statement using a simple construction of a generalized suffix tree which is a suffix tree.

The most occurring substring on a suffix tree is found on a state \( x \neq \text{root} \) that is ancestor to the most leaf states. This is the case because we only create inner states when at-least two sub strings having common prefix. Therefore, the most occurring substring on a suffix tree is found on the transition leading to such state \( x \). This can be easily extracted by traversing the tree bottom-up using depth-first-search and keeping a frequency count on each state.

To extend this approach to a generalized suffix tree, we need to take into account that two sub strings can have a common path and hence traversing this path once, as is the case with a simple suffix tree, does not consider that the prospect of this path occurring twice in the string \( \Omega \). Consequently, we need to traverse the tree multiple times to get the correct frequency count of each state. To get the correct frequency count, we traverse every valid suffix present in \( \Omega \). In that way, we account for the correct frequency count of each state.

Now that we have the frequency count on each state, we can easily extract the most occurring substring of minimum length \( n \) by finding the transitions that lead to the states that occur the most frequent and that are of at least of length \( n \). We can then extract the substring using the start and end index of the transition that leads to these most frequent states. The resulting algorithm is given Algorithm 4. Algorithm 4 can be extended to extract most occurring substrings that occur at least \( r \) times the most occurring substring. The value of \( r \) ranges from 0.1 to 0.9, therefore \( 0.1 \leq r \leq 0.9 \).
Algorithm 5 is the algorithm that extracts most occurring substrings.

Algorithm 4 Most occurring substring of minimum length $n$

1: DFS to set occurrences count for every state
2: node $\leftarrow$ root
3: vector $\leftarrow \{\}$ \> stores a pair of start index and length of substring
4: string_dept $\leftarrow 0$ \> stores the transition depth so far
5: highest_frequency $\leftarrow 0$ \> keeps track of most frequent state
6: procedure MOST_FREQUENT_SUBS(node, vector, string_dept, highest_frequency, n)
7: for $i \in$ node.transitions do
8:    MOST_FREQUENT_SUBS(i, vector, string_dept + i.transition_depth, highest_frequency, n)
9: end for
10: if string_dept $< n$ or (node is leaf state and string_dept == minimum) then
11:    return
12: end if
13: length $\leftarrow$ node.isLeaf ? string_dept $-$ 1 : string_dept
14: if node.occurrence_count == highest_frequency then
15:    vector.push(\{node.end_index - string_dept + 1, length\})
16: end if
17: if node.occurrence_count $>$ highest_frequency then
18:    vector $\leftarrow \{\}$
19:    highest_frequency $\leftarrow$ node.occurrence_count
20:    vector.push(\{node.end_index - string_dept + 1, length\})
21: end if
22: end procedure

4.2.1 Complexity analysis of Algorithm 4 & Algorithm 5

Since both the algorithms are dominated by the same part of the algorithm in terms of complexity, we just analyze the time complexity of one of the algorithms, namely Algorithm 4. We divide the complexity analysis of Algorithm 4 in two following parts.

Line 1

As mentioned earlier in this document that a single traversal of the generalized suffix tree for $\Omega$ does not suffice to set the occurrences count of the states accurately. There are at most $|\Omega| - n$ suffixes in the tree because there are at most $|\Omega| - n$ leaf states. Traversing a suffix of length $m$ in a generalized suffix tree takes at most $m$ steps. Therefore, traversing all $|\Omega| - n$
Algorithm 5 Most occurring substrings of minimum length $n$

1: DFS to set occurrences count for every state
2: node $\leftarrow$ root
3: vector $\leftarrow \{\}$  \hspace{1em} $\triangleright$ stores a pair of start index and length of substring
4: string dept $\leftarrow$ 0  \hspace{1em} $\triangleright$ stores the transition depth so far
5: highest frequency $\leftarrow$ 0  \hspace{1em} $\triangleright$ keeps track of most frequent state
6: procedure Most frequent subs(node, vector, string dept, highest frequency, n, r)
7: for $i \in$ node transitions do
8:   Most frequent subs($i$, vector, string dept + $i$.transition.depth, highest frequency, n, r)
9: end for
10: if string dept < n or (node is leaf state and string dept == minimum) then
11:   return
12: end if
13: length $\leftarrow$ node.isLeaf ? string dept - 1 : string dept
14: if node.occurrence count == highest frequency then
15:   vector.push({node.end index - string dept + 1, length})
16: else if node.occurrence count > highest frequency then
17:   remove all substrings whose occurrence count is less than $r \times$ highest frequency from vector
18:   highest frequency $\leftarrow$ node.occurrence count
19:   vector.push({node.end index - string dept + 1, length})
20: else
21:   if node.occurrence count $\geq r \times$ highest frequency then
22:     vector.push({node.end index - string dept + 1, length})
23: end if
24: end if
25: end procedure
suffixes will take \( m \times (|\Omega| - n) \), where \( m \) is the length longest suffix present in the tree. All in all, the run time complexity of Line 1 is \( O(m \times (|\Omega| - n)) \).

**Line 2 - 25**

The procedure does a simple one pass depth first traversal of the generalized suffix tree of a string \( \Omega \) of length \( n \). Therefore, the run-time complexity of the procedure is the same as the one discussed in Section 4.1.3 which is \( O(4(|\Omega| - n) - 1) \).

All in all, the worst case run-time complexity of the algorithm is \( O(m \times |\Omega|) \).
Chapter 5

Codewords case study

Besides the ASML models to test our hypothesis, we would like to test our hypothesis using smaller Mealy machine that also exhibit the far-out distinction problem. This chapter provides in detail the family of Mealy machines we have come up with that exhibit the far output distinction problem.

5.1 Codewords family

This section will detail how the states and the transitions in the codewords family of Mealy machine are formed. Imagine we are at an arcade and we stumble on this machine that has a finite set of prizes $P$, and in order to win a prize $p \in P$, we must guess the codeword $c \in C$ from the set of codewords $C$ mapped to the prize.

Codeword $c$ for a prize $p$ is a sequence of inputs from $I$. Let $f : P \rightarrow C$ be a bijective function that maps prizes in $P$ to codewords in set $C$. We also impose a condition on the set of codewords such that a codeword of one prize $p$ is not the prefix of another code-word for another prize $p'$. Hence, the set $C$ is a so-called prefix code such that for $b, b' \in P$ : if $f(b)$ is a prefix of $f(b')$ then $b = b'$.

There also exists a special input prize in $I$, that tells us the prizes we have won so far. The outputs of this Mealy machine on every input $i$ other than the special input prize is -. The output for input prize is the set of prizes won so far.

The states of these Mealy machines consists of a tuple $(s, w)$, where $s \subseteq P$ and $w \in I^*$ such that $w$ is a prefix of a word in $C$. The number of states in this Mealy machines can be calculated as follows:

- The different number of subsets of $s$ is power set of $P = 2^{|P|}$.
- Let the number of prefixes of codewords in $C$ including the empty prefix be $q$. 

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• The total number of states is given by \( q.2^{\left| P \right|} \).

Similarly, the number of transitions is calculated as follows:

• The number of states as given above = \( q.2^{\left| P \right|} \).

• The number of inputs is \( \left| I \right| \).

• The total number of transition is \( q.2^{\left| P \right|}.\left| I \right| \).

For this class of Mealy machines, we have \( I = \{0, 1, \ldots, 9\} \cup \{\text{prize}\} \).

Therefore, the number of transitions becomes \( 11q.2^{\left| P \right|} \).

Let \( S \) be the set of states of this Mealy machine. The transitions function \( \delta : S \times I \rightarrow S \) is defined as follows: for \( s \in S \), \( x \in I \), \( b \in P \), \( w \in I^* \) and \( \epsilon \) denotes the empty sequence.

\[
\delta((s, w), x) = \begin{cases} 
(s, wx), & \text{if } wx \text{ is proper prefix of codeword } f(b), \text{ for some } b \in P \\
(s \cup b, \epsilon), & \text{if } f(b) = wx, \text{ for some } b \in P \\
(\emptyset, \epsilon), & \text{if } w = \epsilon \text{ and } x \text{ is not a prefix of any codewords} \\
(s, w), & \text{otherwise}
\end{cases}
\]

5.2 Motivation

In the figure 5.1, we can see a Mealy machine for two prizes, whose code
words are 11 and 22. For the clarity of the figure, \( p \) is used instead of
the special input \( \text{prize} \) and the prizes won are represented in the form of
binary sequence where each index corresponds to each prize, for example
00 represents no prize won and 01 represents second prize won. From the
figure itself, we can make up why this is family of Mealy machines exhibit
the far-output distinction behavior problem for Mealy machine with large
amount of prizes.

The paths of interest in this family of Mealy machines are the paths that
lead to all prize winning state which labeled as 9 in figure 5.1. This state
can be reached via two paths namely, 0-2-3-4-9 and 0-1-6-8-9. The number
of such paths is just the permutation of codewords, where the number of
elements and sample size are equal. Hence, for \( n \) codewords there are \( n! \) such
paths that lead to a winning state. In this small example, the lengths of
these paths and the number of such paths is small that the learning algorithm
can easily learn them. Therefore, since the factorial function grows rapidly,
the active automata learning oracle will have difficulty learning the Mealy
machines of this family with large amount of prizes.
Figure 5.1: A Mealy machine for two prizes with codewords 11 22.
Chapter 6

Experimental Results

This chapter starts with a description of the environment used to carry out the experiments. Furthermore, it presents the results of these experiments.

6.1 Experiment environment

We will first begin by explaining the schematic view of the environment that is used to carry out the experiments. We will test our hypothesis against the codewords case that we introduced in Chapter 5.

Figure 6.1 depicts the environment from top to bottom, starting at the top where codewords for the prizes and a positive integer \( n \) are supplied to a script that generates a model (Mealy machine) (labeled as model in Fig: 6.1) for the respective codewords and a traces file that contains \( n \) traces that end up in a winning state.

We build a generalized suffix tree (GST) using the generated traces file and a positive integer \( m \) which is the minimum length of the most occurring subsequences we want to extract from GST.

We then proceed to feed the most occurring subsequences (SS) and the model to the learning oracle that consists of SUL, learner and the tester. In the experiments conducted, we use the \( L^\# [9] \) learning algorithm.

6.2 Traces for the codewords models

The script that produces the traces file is fed with an integer \( n \) and the codewords. Each trace (TR) is an iteration of a junk (J) trace and a random prize winning trace (RPWT). J traces are valid traces of the model which may or may not win a prize. RPWT are traces for prizes selected at random. The length of a J trace is at most 10 and it is randomly chosen. The total length (TRL) of TR is determined randomly using a random number generator. Let codeword\(_i\) be the \( i^{th} \) codeword and \( x \) be the number of codewords supplied to the script then:
\[
\sum_{i=1}^{x} |\text{codeword}_i| \leq TRL \leq 10 \times \sum_{i=1}^{x} |\text{codeword}_i|
\]

TR may or may not win all the prizes, hence at the end of the TR the traces for remaining prizes that are not won are added.

### 6.3 Experiment setting

We experiment on the models of the codewords in two settings, namely with subsequences (WSS) and without subsequences (WOSS). The difference in both the settings is in the way test suite \( T \) is constructed. The test suite for WOSS is \( T = A \cdot I^{\leq k} \cdot S \cdot C \) and the test suite for WSS is \( T = A \cdot I^{\leq k} \cdot S \). Therefore, a test in the WOSS setting has a prefix from \( A \), first infix from \( I^{\leq k} \), second infix from \( S \) and a suffix from \( C \). The sets \( A, I^{\leq k}, S, C \) are set of access sequences, set of permutations of input symbols up to length \( k \), set of SS and set of characterization sequences \( C \) respectively. We experimented on both settings with \( k = 0 \). We also experimented on WOSS setting with \( k > 0 \) to compare the number of equivalence queries with that of the WSS setting.
6.4 Characteristic of codewords models

We chose the models of codewords to be uniform in terms of the length of each codeword. The choice for this was to reduce the size of the \( S \). We chose the length of codewords \( l \) to be at least 4 and the number of codewords \( c \) to be at least 2. We then gradually increase \( l \) and \( c \) to get more complex models. We wanted these models to be in the sweet spot between not too complex and not too trivial. This was mainly to do with the time frame of this project. As such, the models have \( l \in \{4, 5, 6, 7, 8\} \) and \( c \in \{2, 3, 4\} \).

6.5 Results

In this section, we discuss the results of the experiments we ran. Each experiment consisted of learning the codewords model in WSS and WOSS setting. We set \( k = 0 \) in the WSS setting. The following two discusses the results when \( k = 0 \) and \( k > 0 \) in the WOSS setting.

6.5.1 \( k = 0 \)

For \( k = 0 \), the WOSS setting failed to learn any of the models given in Table 6.1. The WSS setting learned all the models given in Table 6.1 and as such the length of the SS used is reported under column \( s \) of Table 6.1. The SS that are used are presented in Table 6.4. We set \( s \) to be the length of the codewords, so that the GST can extract the codewords that were used to construct the model.

6.5.2 \( k > 0 \)

For \( k > 0 \), we were able to learn some of the smaller models as presented in Table 6.3. Since these models were also learned in the WSS setting, we decided to report the number of membership and equivalence queries that were queried in both the settings as presented in Table 6.3 and Table 6.2. We also reported the number of queries for some of the more complex models in WOSS setting because in comparison with the WSS setting, they were immense. Despite the large number of queries, the WOSS setting still failed to learn the models. Figure 6.2 shows the equivalence queries with respect to the length of codewords in both the settings.

6.6 Conclusion

From the experiments we have conducted, we can say that the knowledge of the most occurring subsequences do definitely help us in learning models that belong to the codewords family. Lastly, we conclude that the addition of the most occurring input subsequences in to the test sequences definitely
help us reach states that were not reached prior to the inclusion of these sequences. Setting $s = l$ does extract all the codewords of a said model from the GST. Therefore, the findings of the experiment conducted do claim in support of our hypothesis which states that the inclusion of the most occurring subsequences do help with learning models that exhibit the far output distinction behavior.

<table>
<thead>
<tr>
<th>Model name</th>
<th># states</th>
<th># traces</th>
<th>l</th>
<th>c</th>
<th>codewords</th>
<th>s</th>
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<td>28</td>
<td>20</td>
<td>4</td>
<td>2</td>
<td>1234 5678</td>
<td>4</td>
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<td>4</td>
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<td>1234 5678 9438</td>
<td>4</td>
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<td>75</td>
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<td>2</td>
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<td>5</td>
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<td>5.3.50</td>
<td>103</td>
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<td>5</td>
<td>3</td>
<td>12345 67892 43716</td>
<td>5</td>
</tr>
<tr>
<td>5.4.75</td>
<td>271</td>
<td>75</td>
<td>5</td>
<td>4</td>
<td>12345 67892 43716 51794</td>
<td>5</td>
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<td>6</td>
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<td>6</td>
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<td>2</td>
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<td>9</td>
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</table>

Table 6.1: Codewords models for experiment

<table>
<thead>
<tr>
<th>Model name</th>
<th>Membership queries</th>
<th>Equivalence queries</th>
<th>Learned</th>
</tr>
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<tbody>
<tr>
<td>4.2.20</td>
<td>980</td>
<td>115</td>
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<td>4.3.50</td>
<td>3921</td>
<td>4963</td>
<td>Yes</td>
</tr>
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<td>1642</td>
<td>41</td>
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<td>174</td>
<td>Yes</td>
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<td>2142</td>
<td>Yes</td>
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<tr>
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<td>3826</td>
<td>240</td>
<td>Yes</td>
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<tr>
<td>9.2.20</td>
<td>4458</td>
<td>485</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 6.2: Membership & Equivalence queries in WSS setting
<table>
<thead>
<tr>
<th>Model name</th>
<th>Membership queries</th>
<th>Equivalence queries</th>
<th>$k$</th>
<th>Learned</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.20</td>
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<td>6859</td>
<td>2</td>
<td>Yes</td>
</tr>
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<td>41715</td>
<td>2</td>
<td>Yes</td>
</tr>
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<td>2</td>
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<td>1193855</td>
<td>3</td>
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<td>No</td>
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<td>613348</td>
<td>3</td>
<td>No</td>
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<td>No</td>
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<td>2508</td>
<td>59770207</td>
<td>5</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 6.3: Membership & Equivalence queries in WOSS setting with $k > 0$

Figure 6.2: Equivalence queries with respect to length of codewords
<table>
<thead>
<tr>
<th>Model</th>
<th>Most occurring subsequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>4_2_20</td>
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</tr>
<tr>
<td>4_3_50</td>
<td>1234 12345 81234 2345 234567 2345678 34567 345678 4567 45678 9438 5678</td>
</tr>
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<td>12345 23456 212345 78921 789212345 67892 89212345 9212345</td>
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<tr>
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<tr>
<td></td>
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</tr>
</tbody>
</table>

Table 6.4: Most occurring subsequences
Chapter 7

Conclusions & Future work

In conclusion, we explored the idea of extending test sequences with the most occurring subsequences to help us learn models that exhibit the far output distinction behavior. We defined and specified a family of Mealy machines called codewords that exhibit the far output distinction behavior. We obtained the set of most occurring subsequences by building a generalized suffix tree of the input sequences that we obtain from the logs of these models. For the codewords family of Mealy machine, we generated valid traces of the models which were used to generate the generalized suffix tree. We used an extension of Ukkonen’s algorithm, proposed by Gusfield [5], to build a generalized suffix tree. We ran experiments on the codewords family of Mealy machine in two settings. In the first setting, we tried to learn the models without the inclusion of the most occurring subsequences in the test sequences. Conversely, in the second setting we included the most occurring subsequences in test sequences. We found the second setting required much fewer queries to learn the models. Therefore, we conclude that the inclusion of the most occurring subsequences in test sequences help with learning of models that exhibit the far output distinction behavior.

7.1 Future works

In this thesis, we experimented our method with the codewords family of Mealy machines. The number of input symbols in these models is capped at 10. This makes them easier to experiment with it compared to the ASML models. Moreover, the knowledge of long input sequences that cause the far output distinction behavior gives an upper-hand in finding appropriate candidates for the length of subsequences to utilize. However, this is not the case with the ASML models. The application of our method to learn the ASML models can possibly help with learning of the model or help expose new behavior that these models exhibit.
Bibliography


