INTONATION AND TEMPERAMENT: A SPECIAL ORCHESTRAL REHEARSAL

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Abstract. This paper describes aspects of the practice of intonation when playing music. The paper takes the form of a fictional story about a rehearsal of an amateur orchestra in which special attention is given to intonation and temperament. Through a couple of examples and demonstrations the members of the orchestra learn what playing 'in tune' means and what to listen for when they try to play in tune. It turns out that playing in tune is not as easy as it seems, a lot of music theoretical concepts are involved.

1. Introduction

At the beginning of what seemed like a normal orchestral rehearsal, the conductor said to the orchestra: "Today, we will have a special orchestral rehearsal where we will give a lot of attention to the intonation". Some of the members of the orchestra sighed lightly, because intonation was not a very easy subject to study, and furthermore, they knew the conductor and his preference for - sometimes - difficult mathematical explanations of intonation phenomena. "Prepare for a long evening", somebody said. The conductor did not get distracted too much by these reactions, after all, there were enough people, like for example the timpani player[1], who *did* enjoy talking about intonation and music theoretical topics.

2. Beats

"Before we start playing", the conductor said, "let us play some scales together to warm up a bit and let us concentrate especially on the intonation of the notes we are playing. They started with a scale in C major over 3 octaves. The conductor divided the orchestra in two halves. One half started playing the scale on C, and other half started to play the same scale two notes later, when the first half had arrived on the major third of the scale. This sounded nice and harmonious, "but, it could be done better", was the opinion of the conductor. "Does everybody know what to listen for, when I ask you to

[1]Besides the fact that Henk Barendregt, whose 60th birthday is celebrated with this volume, was the author's PhD supervisor, they have also played together for many years in the VU orchestra, where Henk Barendregt played the timpani and the author played the violin. Memories of the orchestra served as inspiration for this paper.

REFLECTIONS ON TYPE THEORY, λ-CALCULUS, AND THE MIND
Essays Dedicated to Henk Barendregt on the Occasion of his 60th Birthday

focus on the intonation?", the conductor asked. Some nodded and others shook their head, but when the conductor asked for the answer to the question, most people did not know what to say ... except for the timpani player of course. He said: "I think one can listen for the beats, when trying to play in tune". Some others reacted on this, since they once heard their piano tuner talking about beats. "Indeed, we can listen for beats", said the conductor. "Let's leave the beats in piano sound out for a moment, we can come back to that later, that is a bit of a different topic". "Let us listen to the beats of a sound so that everybody knows what to listen for. Hmm, which sound shall we choose.. o yes, the clarinets would be perfect for this experiment. Clarinets, could you please both produce an A, and try to be in tune with each other", the conductor asked. The two clarinetists played an A and everybody listened. "Do we hear any beats here?", the conductor asked. No, there were no beats present. "Now one of you keeps the A as it is now, and the other should produce a sound which is just a little bit higher than the first A, so just squeeze together your lips just a bit". Not an easy exercise for the clarinetists, but they managed very well. A sound appeared in the rehearsal hall which was the same sound as before, but it seemed to alternate between a loud and soft sound. Some people seemed to think the clarinetists did this on purpose, but then the timpani player said: "Yes, these are beats". The clarinetist who was producing the higher tone got the idea of the exercise quite quickly, and he started experimenting what would happen if he would squeeze his lips even more and produce a higher tone. "Wait a second!", the conductor said, "I want everybody to understand what is happening, or perhaps .. the timpani player wants to explain this". "O yes, I would like to explain, that's okay", the timpani player said. "What we hear here, is a physical phenomena. If two tones that are played simultaneously have not exactly the same frequency, we can hear beating, which is a slow amplitude variation. That is what we just heard. If the tones differ even more, the beating becomes faster, that is, the amplitude will vary more frequently [8, 2, 14, 7]. If the two tones then vary even more, the beating disappears and then two different tones can be noticed, instead of only one with varying amplitude, as we heard before. The point where we stop hearing one beating tone and start to hear two different tones is called the critical bandwidth" [12, 10, 13]. The timpani player wanted to go on with his explanation, but the conductor interrupted him: "I think that's enough information for now, let us try if everybody can hear the process that you have just described", and he asked one of the clarinets players to keep a constant A, and the other to start playing the same A, and then squeezing his lips until the tone he produced was about a semitone higher, and then going back. The clarinets tried to do what was asked to them, but this experiment turned out to be quite difficult. "Ah, well perhaps I should have asked two string players for this experiment in the first place, on a violin this will be much easier because a sliding finger is probably easier to control than the tension in your lips". He asked two violinists to do the same thing as he asked the clarinetists before. One violinist could just play an open string, and the other could slide very slowly with one finger until the point where the tones differed more than a semitone. Some oh's, and ah's sounded from the orchestra because most of them could now very well hear the process that was explained by the timpani player. Except for the critical bandwidth, that turned out quite hard to hear, to most people it was not quite clear when the sound was just a very harsh sound of a very fast beating, or it was actually the harsh sound of two tones that did not sound well together at all. "That is indeed very hard to hear", said the conductor. "But the point of this exercise was to make you aware of the beats, so that we now have something to listen for if we try to play in tune. What we will try to do is to minimize the beats. We can also

minimize the beating if not playing the same note but two different notes", the conductor went on. "Let's try this with a perfect fifth", and he asked the same two violinists who had just illustrated the beats to illustrate the same process when playing a perfect fifth. "So one of you should play an open D string and the other will play an A with one finger on the D string, so to be able to slowly slide to the A and a bit over, so that everybody can hear the beats and the absence of the beats when your finger reaches the position of the A that is in tune with the D." The violinists did what was asked of them and illustrated the process very nicely. One of the viola players however said: "but this is very difficult to listen for when we play a scale like we started with. How should we ever manage to hear this beating if we are playing fast?" But the conductor smiled and said: "Yes, you are right, it is difficult, and therefore you should all practice this at home, and perhaps we can even have extra sectional rehearsals for this purpose". And with this last remark, under protest of many people, he closed the experiment and they went on with playing the scales they started the rehearsal with.

3. Combination tones

After some more playing, one of the violinists said to the conductor: "My violin teacher taught me to listen to combination tones when I practice intonation on double stops, that is perhaps an other way to determine if a sound is in tune or not." Some other violinists nodded, they had some experience with this as well. "What a good idea", the conductor admitted. "Let us try to hear some combination tones as well. Who knows and can explain to the others the idea behind combination tones?" In an automatic answer all faces turned toward the timpani player, everybody knew that he would know the answer and enjoyed explaining this to the rest of the people. "O well, because you insist", he started. "Two tones with a frequency difference that is not too large can instantiate the perception of combination tones [8, 11, 16, 6]. We can distinguish sum tones and difference tones, but the difference tones are generally easier to hear, so let's concentrate on that. Perhaps we should first try to listen to a difference tone, before I continue with my explanation. Why don't we have two flutes playing the interval of a perfect fifth, one plays a C and the other plays a G a fifth above the C. The difference tone that we should here when they play in tune is a C one octave below the C that is played." The two flutists played the tones and everybody tried to listen for the low C. Some whispered that they heard it, other looked confused, and some tried to play the low C on their own instrument very soft, so that they would have an idea what to listen for. "So, this is also a matter of practice", the timpani player said, "but it can be learned fairly quickly, in my opinion". The violinists who had indicated before that they practiced with this method nodded in agreement. "It is easy to know which difference tone should be heard if you place the notes in a common harmonic series", the timpani player went on explaining. "You can just take the difference of their harmonic number to know the harmonic number of the difference tone. For example, the tones that were just played belonged to the harmonic series on C, the C that was played was the fourth harmonic and the G the sixth harmonic, $6 - 4 = 2$ so the difference tone we heard was the second harmonic in that series on C, which is a C one octave above the fundamental and one octave below the C that was played." Some members of the orchestra started to whisper and ask each other questions because they did not really know exactly what an harmonic series is. The timpani player promised that he would explain that in the coffee break to the people who where interested to hear about the harmonic series, and he

said he could even advise some literature [2, 7, 8, 5]. Especially many string players seemed interested to learn about combination tones, since many of them realized that they have always used this principle when tuning their instrument, but they did not know that the sound they listened for, was actually a difference tone.

4. Syntonic comma

"So, we have done enough warming up exercise now, I would think. Let's start at the beginning of Brahms", said the conductor. They had been rehearsing Brahms's fourth symphony for a short period only, the concert at which the piece would be performed was in a couple of months. They started at the beginning of the symphony and played tutti for a while, just like they were used to doing at normal rehearsals.

Then the conductor stopped them, and wanted to focus on the string section for a while, but he asked the wind players to listen carefully, because this was of interest for them too. The passage the strings had to play many times, was a very difficult passage for intonation. "Does everybody know how he or she has to tune the notes that he's playing?", the conductor asked the orchestra. "Well, we have to listen for beats and combination tones, as we've just practiced, don't we?", somebody said. "Yes, that's correct, but does everybody know which note to focus on, when he tunes his own? In other words, to which note do you tune your own note?" This was again something knew, and most people did not really understand which answer the conductor expected. The conductor smiled, happy that he could do another demonstration, and said: "We will give a little demonstration with the violin players sitting on the first row, to hear what this passage is all about", the conductor announced. The people he had pointed at looked a bit alarmed since they had no clue what was expected of them, but the conductor explained: "First, I need to hear an open G string". One of the violins played an open G. "Then, a D tuned at a perfect fifth, so please don't use the open D string here, but play a D on the G string", and a D tuned as a perfect fifth was played as well. "Now, please a perfect fifth on the D, so an A, but again, not an open string please", and one of the violins played an A. "Now, we don't need the D anymore, so you can stop playing", he said to the boy who was playing the D. "But we do need the G and the A, so please remember exactly where your finger is on the string". For the girl who was playing the G that was easy, because she was playing an open string. "I need two more people now, each to play an E", and he pointed at two violin players. "The person who played the G can stop for a moment. I need now somebody to play an E, tuned as a perfect fourth below the sounding A." When everybody heard the E, the conductor said: "Okay, please remember where your finger is on the string to produce this E, since I need you to play it once more in a moment". "Now I need to hear another E, this time tuned as a major sixth above the G", and with a gesture with his hand, he asked the person who played the open G string the first time, to start playing again. The second E was sounding. "Can I hear the first E again?" the conductor asked. The first E was played again, and there was already some surprise noticeable on the faces of the other members of the orchestra. "Please alternate the two E's, so that everybody can hear that they are clearly not the same", the conductor said, and then more and more people heard it. For most people, it just seemed to be a complicated trick, and they were not totally aware what was going on. But some violin players showed some recognition. One said "hey, isn't this the same thing as my violin teacher sometimes points out to me, when I have to play a G, E and A in one and the same chord or two chords close together, which is for example

in the Chaconne of Bach where in bar 139 an E is tuned against the open G string and in the next bar the E is tuned against the open A string[2]?" Some people laughed, since they clearly couldn't believe that the person who said this had ever played a piece as difficult as Bach's Chaconne, but the conductor smiled and said: "Very well, that is indeed what I'm talking about. And since violins are tuned in perfect fifths, it is quite impossible to play an E with the first finger on the D string such that it is in tune with both the open G string and the open A string." Now a lot more people understood what the conductor was talking about and some violin and viola players started trying out on their own violin what the conductor had just explained. "Who knows what the difference between these two E's is called?", the conductor asked. It appeared to be quiet for some time, but of course the timpani player knew the answer: "It's the syntonic comma", he said [7, 2, 1].

"Let's do a quick second experiment", said the conductor who now got really enthusiastic. "This will illustrate the same thing. I need a low C, why don't the celli play that for me", and the celli played their open C string. "Very well", said the conductor, "and now a G tuned to the low C of the celli, violas can you play that? Very good. The celli can stop playing now, and I want the second violins to play a D a perfect fifth above the G. The violas can stop playing as well, and then the first violins can play an A a perfect fifth above the D. The second violins can stop playing their D and can instead tune an E a perfect fifth above the sounding A." All this was done very carefully, and the conductor pointed out frequently that it was important to listen very well. "Now go on with playing the E and the celli can join in with their low C again. The question now is: is this in tune?" The low C and the E of the violins made some kind of a major third, but is was not in tune at all, everybody could hear that. Then the conductor asked if the violins could change the E such that it was in tune with the C, and the violins who were playing the E, lowered the tone such that is was in tune with the C. "Is the difference between the two E's the syntonic comma again?", one of the double bass players asked. The conductor confirmed this. "Yes it is, and this is why I always ask the string players at the beginning of a rehearsal when they tune to the A of the hobo, to tune the fifths a bit narrow." This made sense to many of them, since they have been aware of this for some time, the E strings of the violins of course have to be in tune with the C strings of the celli. "But", said a violin player, "do I now understand correctly that it is impossible to play everything in tune? If I tune my E string to the C string of the celli, the E is out of tune with my own A string!". Other people joined this criticism and the whole atmosphere suddenly turned a bit negative. After all, first more and more people got enthusiastic at the idea of now really understanding the idea of playing in tune and being able to hear what is in tune and what not, and now suddenly it turned out to be impossible to play in tune after all. That was a bit disappointing for many people. "Well, yes", said the conductor, "indeed tuning is often a compromise, especially when playing polyphonic music, like we do here. And for that reason, the equal temperament was invented, which we know as the tuning of a piano. This is perhaps also a good moment to have our coffee break, then we can proceed after the break with the other piece we are rehearsing, in which also a piano participates, which is tuned in equal temperament." The piano was not just participating in the piece which was on the rehearsal scheme, in fact, they were to play a piano concerto, so the piano played a leading role.

[2]The author wants to thank Nick Devons for pointing out this example.

5. Equal temperament

The coffee break lasted a bit longer than usual since both the conductor and the timpani player were explaining a lot of things about intonation and temperament to the interested musicians. Finally, they all sat down again and the second half of the rehearsal started. "The first half of the rehearsal we tried to play in just intonation, that is, to play in tune so as to minimize the beats and to maximize the perception of difference tones, just as we practiced", the conductor started. "In the third piano concerto of Rachmaninov, the intonation will be a bit different since we have to play together with a piano. In the passages where the piano participates and plays a dominant role, the rest of the orchestra should try to mimic the intonation that the piano uses, which is equal temperament. Mr. timpani player", the conductor went on, "since we have been leading this rehearsal kind of together today, would you mind telling the orchestra a bit about equal temperament?" "It is my pleasure", the timpani player replied, and he started. "The syntonic comma, the difference between the two different E's that we heard before the coffee break, is a sign that pure fifths and pure thirds cannot be reunited in one intonation system. Either we had five pure fifths resulting in an E, or we had a pure major third C-E which gave rise to a different E. With most of the instruments in this orchestra we can produce these two different E's, but we can understand that on a fixed tone instrument like a piano, the key that represents the E can only be tuned in one possible way. Therefore, the E on a piano will represent a compromise between the different E's that we just heard. To come to the point of equal temperament", the timpani player went on, "it turns out that if the octave is divided into 12 equal pieces, these notes represent good approximations to the pure intervals. The temperament is therefore called 12 tone equal temperament" [1]. "This makes sense", somebody added", because when I play a chromatic scale, I play indeed 12 semitones which all correspond to the notes on a piano within one octave." Somebody else asked:"There was something said about beats in piano sound earlier, what was that all about?" "O yes, it's good that you remind me", the conductor took over the conversation again. "When tuning a piano, a piano tuner makes use of the beats that the different intervals produce. Since all the notes on a piano are a little bit out of tune compared to just intonation, that is the pure intonation we talked about before the break, the intervals on a piano will produce slow beats. By counting these beats, a piano tuner can tune the piano in equal temperament."

"What about the gamelan?", the timpani player suddenly asked. "I have always been very interested in the gamelan. Is this instrument tuned in equal temperament as well, do you know?" The conductor looked a bit confused and hesitated to answer, but one of the violin players, who once did a study on the gamelan for her musicology degree could answer this question. "Well", she said, "you could say indeed that one of the possible gamelan tunings is close to equal temperament, but that would be 5 tone equal temperament in this case and not 12 tone equal temperament which we know from our piano. I could give you some literature on this if you like? [9, 3, 15]" Everybody laughed, because this was the first time that somebody had some literature advise for the timpani player, instead of the other way around. "Yes, that would be nice, thank you", said the timpani player, who was always open for new challenges and liked to learn new things, in reply.

"Okay, let's now go back to the rehearsal again", said the conductor who did like that many people got so enthusiastic, but who wanted to limit the more off-topic questions. They did a little exercise to try to hear the difference between equal temperament and just intonation, and the conductor encouraged them do to this at home as well if they had a

piano, because since they had been talking so long, there wasn't much time left for the rest of the rehearsal. "It is most easy to hear the difference if you listen to major and minor thirds. The pure thirds differ more from the equal tempered ones than for example is the case with the fifths", the conductor said. The pianist, who came especially for tonights rehearsal of the piano concerto yawned and said: "I have never heard the difference between just intonation and equal temperament. Many people tried to show me, but I guess I'm just too much used to the sound of a piano." Some people glanced at him as if they were very irritated with this remark but nobody said anything.

"Let us start playing tutti at the beginning of the first movement in a very slow tempo, and try to listen to the piano and tune to the piano as well", the conductor said, and they started playing. "Very well!" the conductor smiled after he gave a sign to stop playing, "you are really starting to play with your ears open, I'm very positive about the progress". Most people were happy as well. Just a few people looked as if they didn't really understand why the conductor praised them so much, but they were happy anyway.

6. Final remarks

They rehearsed the same passage a couple of times more, got some directions about the intonation, and finally played the whole piece once through. At the end of the rehearsal the conductor said: "I am very happy that we have been able to do these exercises on intonation. I think it was useful and it will help you to play better in tune, especially if you also practice this at home". He smiled at this last remark because it was always difficult to get the musicians to study at home as well, no matter how passionate everybody was about music. "Intonation is not an easy subject", he went on, "and as we noticed tonight, it is not always clear which note to pick as the reference note to tune your own note to. There is no simple unambiguous rule system for this [7], although some violin teachers teach a rule-of-thumb system of how to tune sharps and flats [4], as some of you may know. The intonation of a tone depends on the function of the tone in the music, on whether or not other instruments are playing at the same time, and even on style and the character of the music. Some people have a strong intuition and opinion about how a tone should be played, but in general there is no right or wrong here. In many cases, it is sufficient to keep your ears open and try to tune to just intonation, as we practiced. But in case of playing together with a piano, often we should stick to equal temperament. And, when there is confusion what to listen for, I will tell you what to do", he concluded with a smile.

Some people were disappointed about the announcement of the absence of a rule system for intonation, others however got enthusiastic about this unresolved question in the field of music research. The timpani player of course belonged to the second group, and that evening in the cafe - where they always had a drink after a rehearsal - they talked for quite some time about intonation and temperaments.

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