A comparison of time-memory trade-off attacks on stream ciphers

AfricaCrypt 2013

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The Model

When inverting a “random” function $f(x) = y$, with $x, y \in N$

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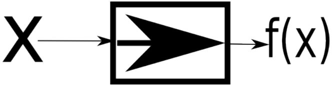
TMTO attacks lie inbetween these extremes

The Model II

In $f(x) = y$, f can be:

- a hash function
- a block cipher $f(x) = f'(x, m)$
- a stream cipher $f'(x) = f(x) \oplus m$

General TMTO



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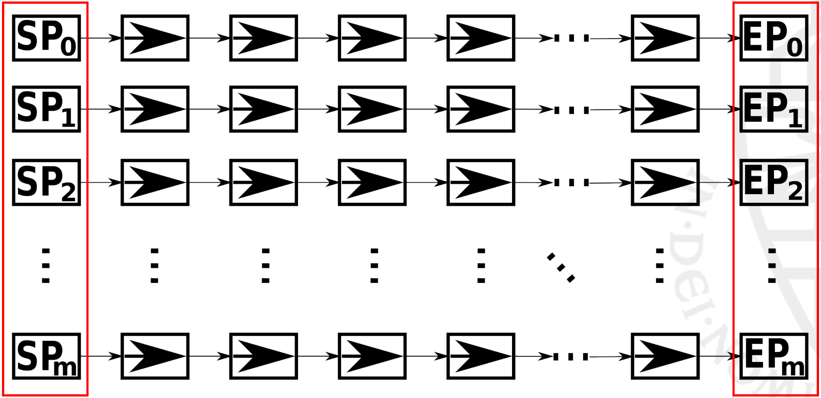

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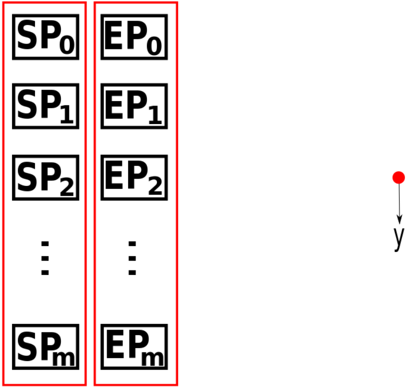

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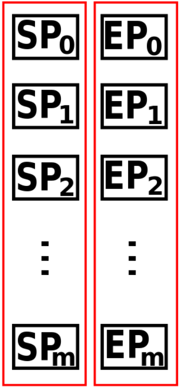

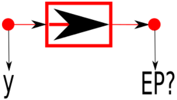

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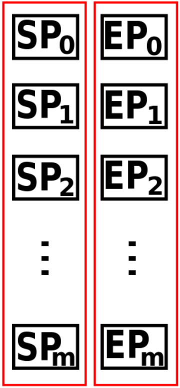

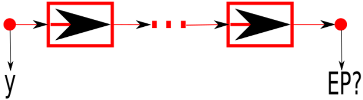

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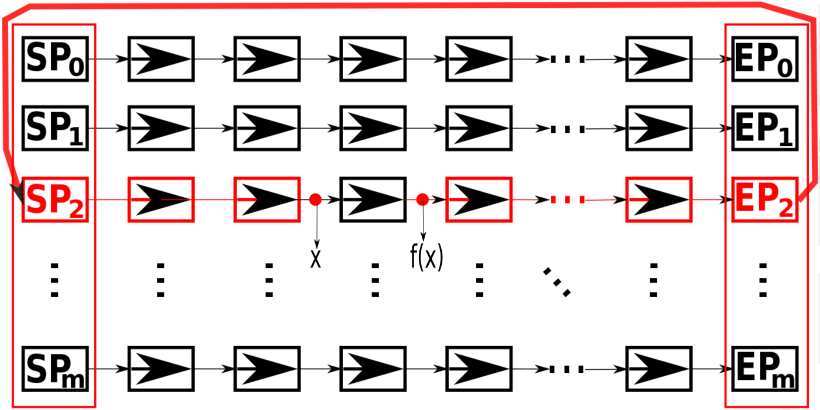
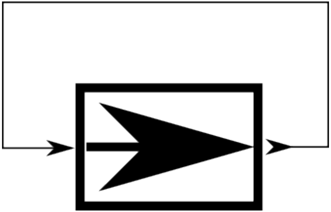
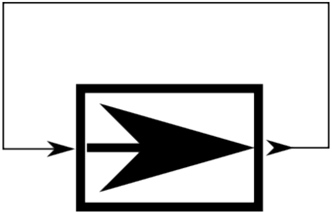


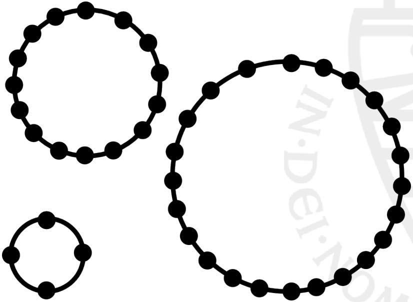
Coverage



Coverage

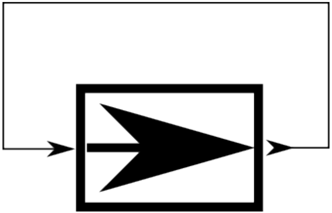
Permutation?

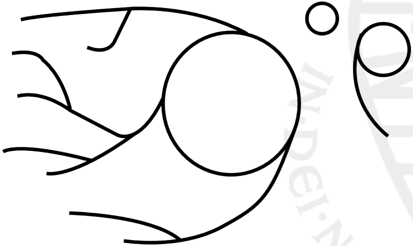




Coverage

A random iterative function





General TMTO: Another problem

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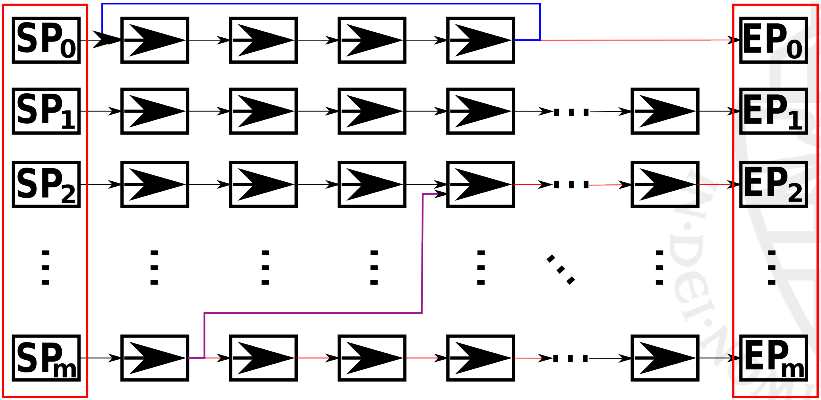
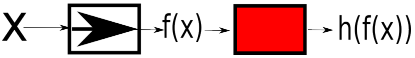


Hellman’s solution



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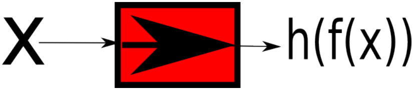

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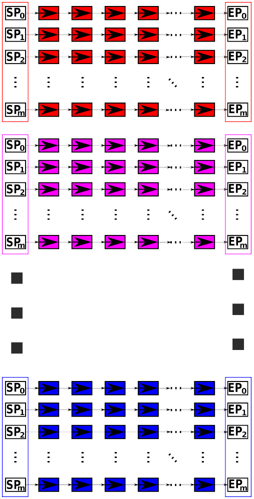

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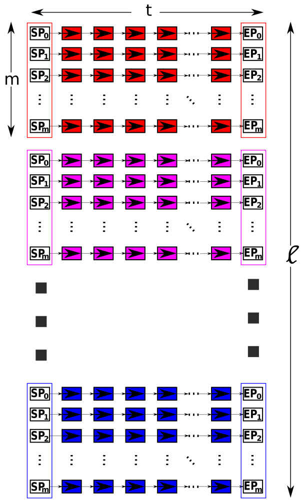

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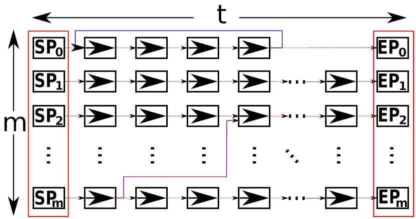


Succes chance bounded by:

$$(1/N)\sum_{i=1}^{m}\sum_{j=0}^{t-1}[(N-it)/N)]^{j+1} \leq \mathbb{P} \leq (mt/N)$$

Proven by Hellman using the matrix stopping rule: $mt^2 = N$

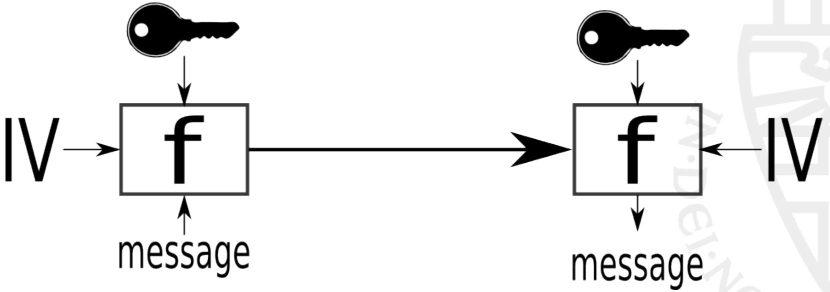
Hellman's solution

For $N = 2^n$ and $mt^2 = N$: Hellman needs 2^n pre-computation encryptions, stored in $2^{2n/3}$ values
$2^{2n/3}$ encryptions then reverse the function f with success chance ≈ 0.55

TMTO Improvements

TMTO Improvements: for Streamciphers

For a block cipher:



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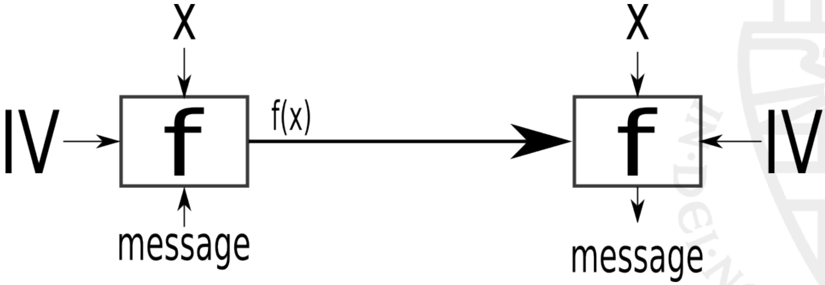

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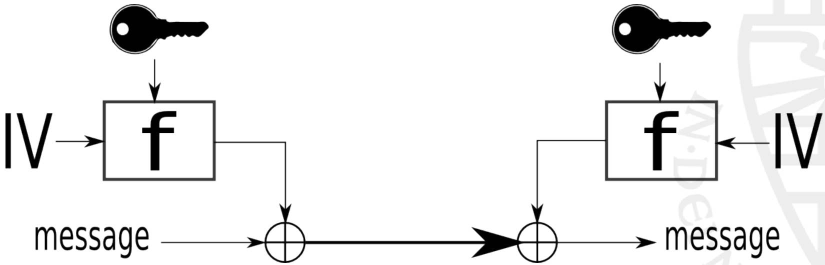

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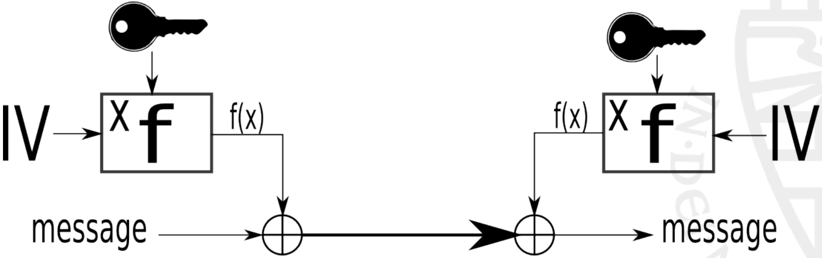

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Even better:

Suppose you created TMTO tables for $|y| = 6$
And you obtain 9 bits:
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Suppose you created TMTO tables for $|y| = 6$
And you obtain 9 bits:
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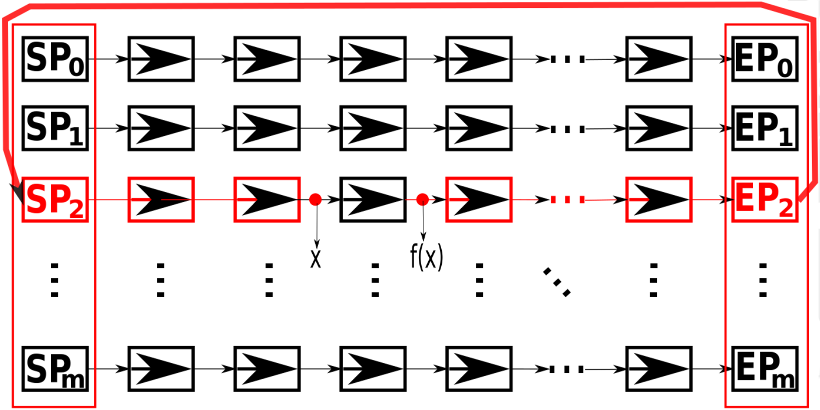
That's 4 samples:
001101
011010
110101
101011

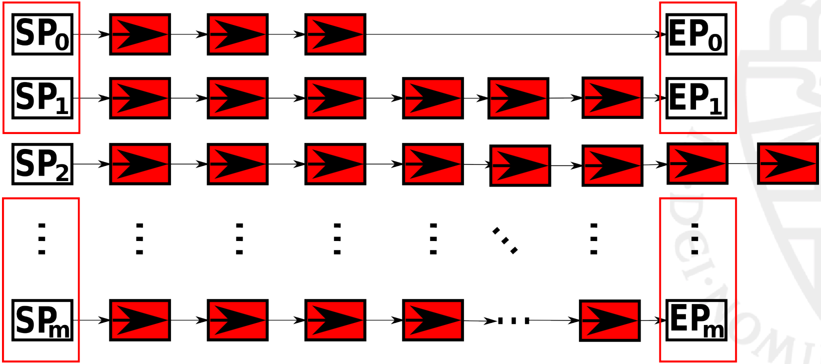
TMTO Improvements: Distinguished Points

Problem: Hellman's attack needs *t* diskseeks per sample per table

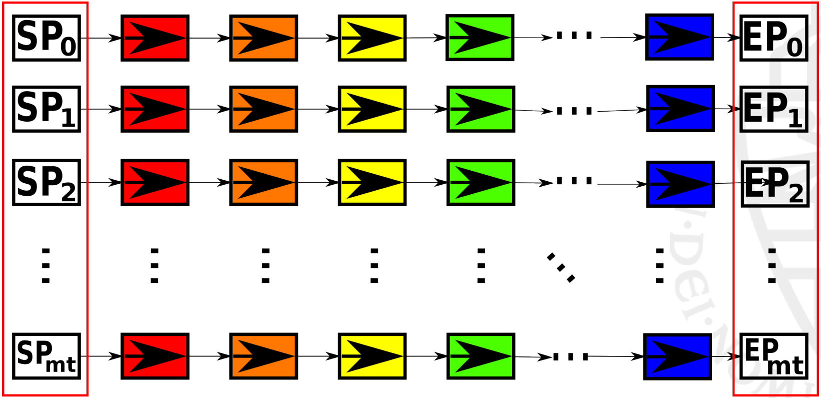


TMTO Improvements: Distinguished Points

End chains in distinguished points (e.g. $0^k||x$)



TMTO Improvements: Rainbow Tables



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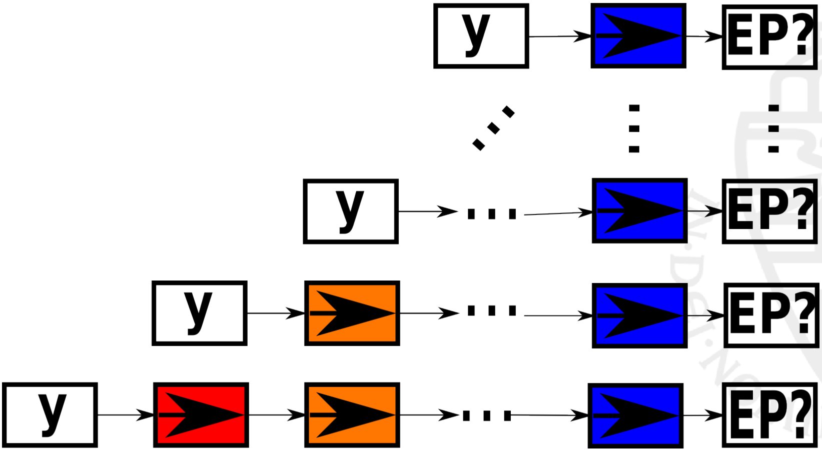

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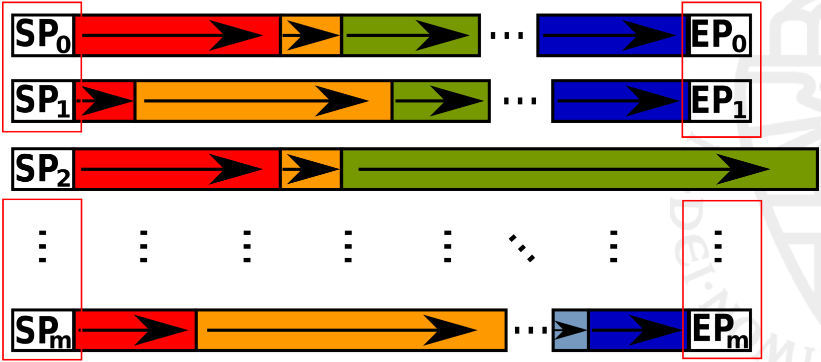

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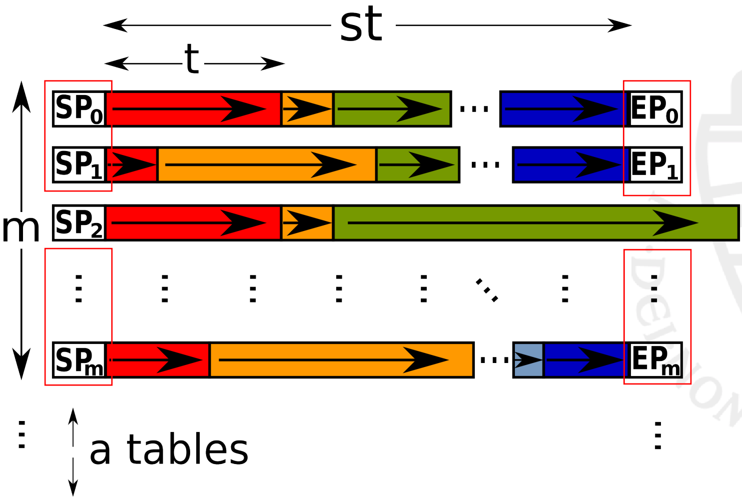

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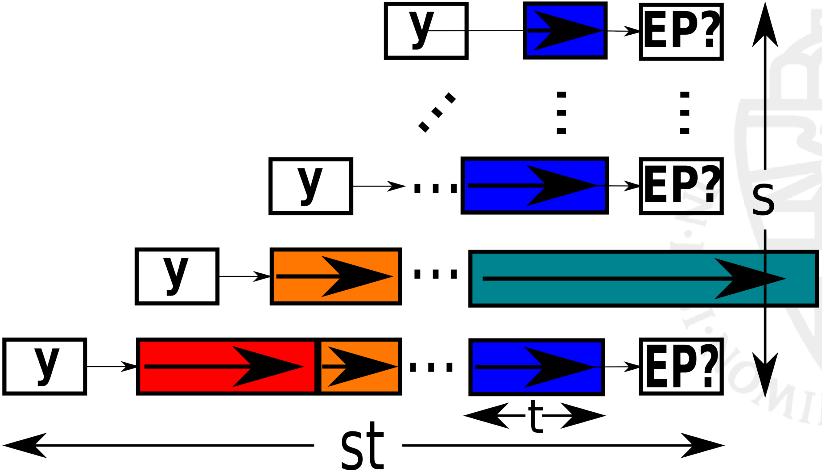
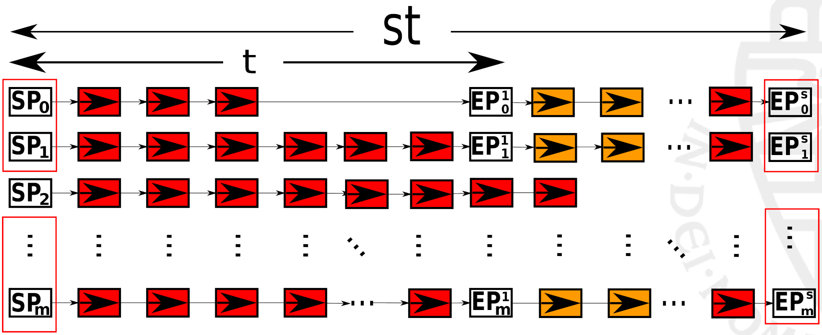


Fuzzy rainbowtables

Essentially an extra Time-Memory trade-off within a TMTO

Fuzzy rainbowtables



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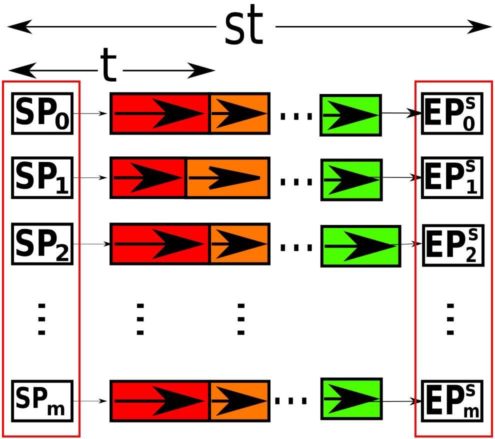


Comparing TMTOs

- $TM^2 = N^2$
- $TM^2 D^2 = N^2$

Comparing TMTOs

What to measure w.r.t. N?

- Pre-computation costs, P
- Memory costs, M
- Attack time costs, T
  - Computation costs, T_c
  - Seek time costs, T_s
- Coverage, C
- Pre-computation ratio, ρ
- Success chance, $\mathbb{P}$

Comparing TMTOs

You might assume $\mathbb{P} = \rho = \frac{C}{N}$

Comparing TMTOs

However: $\mathbb{P} \neq \rho \neq \frac{C}{N}$

- Chain mergers
- Multiple samples
- Not all outcomes of $f(X)$ need to be equally likely

Comparison

For $D\rho = 1$ and $mt^2 = n$:

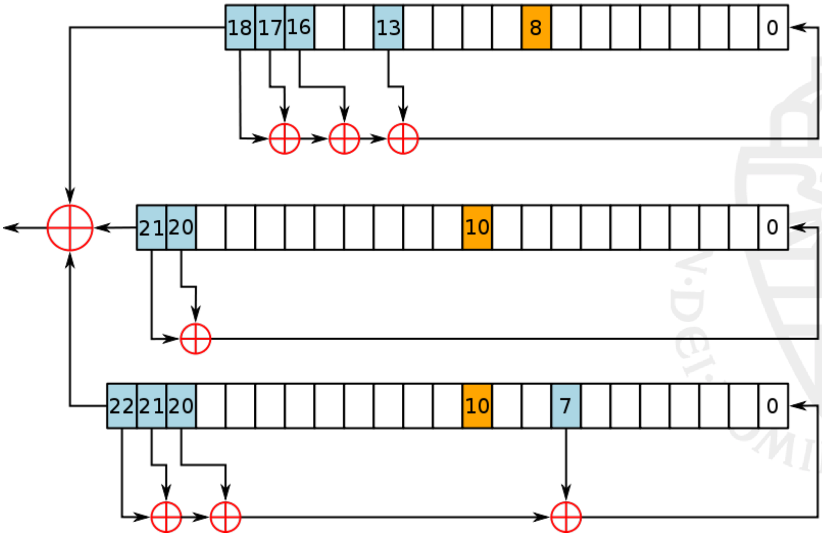
TMTO technique	M	T_c	T_s
Hellman's attack	$2mt/D$	t^2	t^2 in m entries
Dist. Point	$2mt/D$	t^2	t in m entries
Rainbow Table	$2mt/D$	$\frac{t(t+1)}{2}D$	tD in mt/D entries
Fuzzy Rainbow	$2mt/sD$	$\frac{(s+1)}{2}t^2$	t in m entries

But this comparison is unfair

- No measure on chain mergers
- False alarms
- Perfect / non-perfect tables
- What value to choose for s?

A5/1

A5/1



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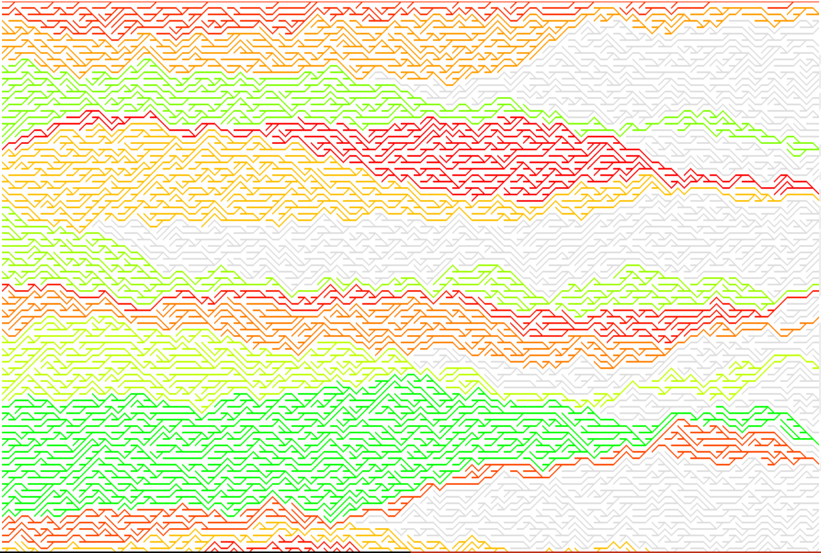

Preparing the attack

- Should fit in 2TB
- Should accept 64 bit keystream samples

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Cipher mode complete

01 01 08 06 32 2b 2b 2b 2b 2b 2b 2b 2b 2b 2b 2b 2b 2b 2b 2b 2b 2b 2b

The Kraken Numbers

- 12 bit distinguished points, $k = 12$
- 8 colors, $s = 8$
- 40 tables (in 1.6TB), $l = 40$
- 8662000000 rows per table ($\approx 2^{33}$)
- In total covers around $2^{53.3}$
- Attack can run ≤ 1 minute

Making perfect tables

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- now throw away chains ending in the same endpoint

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- 2^{33} rows per table
- now throw away chains ending in the same endpoint
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- 29% of all chains merged.
- The perfect tables cover around $2^{52.8}$
- $\mathbb{P} \approx 0.2$

Independent line of work

Hong et al. find the fuzzy rainbowtable approach better for most cases and in their comparison account for chain mergers.
Using $mt^2s \approx N$ as matrix stopping rule.

- Jin Hong and Sunghwan Moon, "A Comparison of Cryptanalytic Tradeoff Algorithms", ePrint 2012-09.
- Byoung-Il Kim and Jin Hong, "Analysis of the Non-Perfect Table Fuzzy Rainbow Tradeoff", ACISP 2013

Questions