Breaking and Fixing Cryptophia's Short Combiner

Bart Mennink and Bart Preneel

KU Leuven

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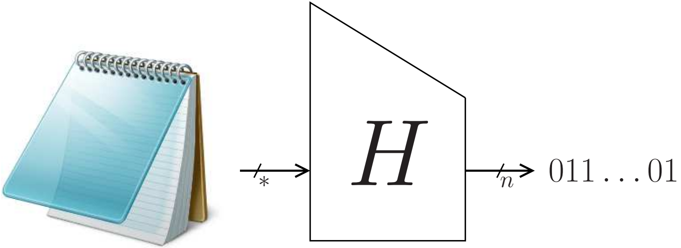
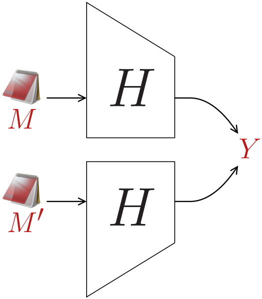
Cryptographic Hash Functions



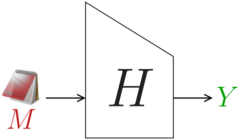
Classical Security Requirements

Collision



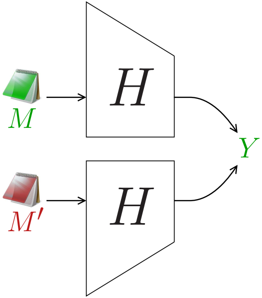
Find $M \neq M'$

Preimage



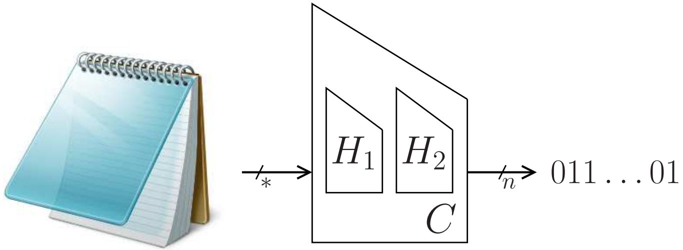
Given Y, find M

Second Preimage



Given M, find $M' \neq M$

Hash Function Combiners



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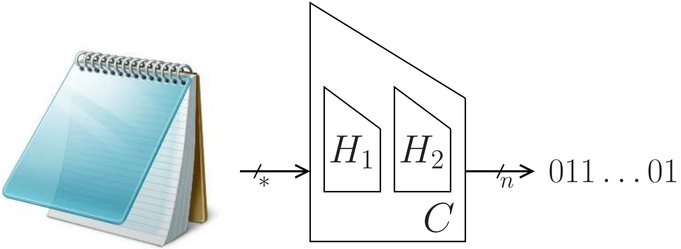

- Extra security barrier: secure even if one hash function is broken
- Applications in TLS and SSL

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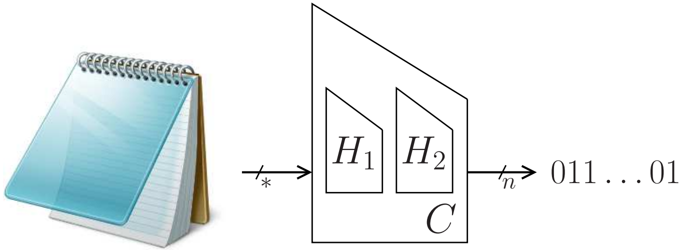

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$$C_{\text{concat}}^{H_1,H_2}(M) = H_1(M) \parallel H_2(M)$$
$$C_{\text{xor}}^{H_1,H_2}(M) = H_1(M) \oplus H_2(M)$$

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Long Output

- Collision robustness: $\approx 2n$-bit output [Pietrzak-C08]
- "Robustness" requires explicit reduction

Ideal Combiner Model [Mittelbach-ACNS13]

- H_1, H_2 based on random oracle
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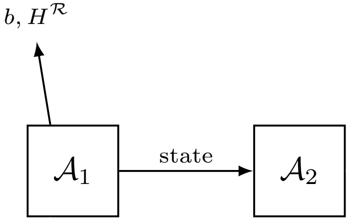

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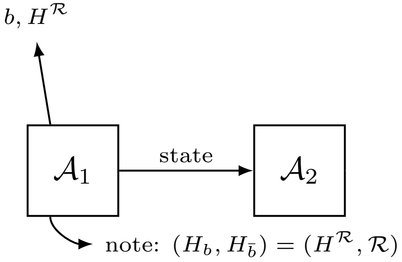

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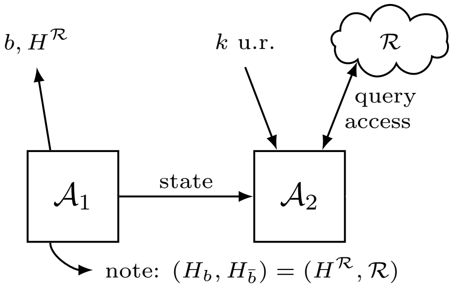

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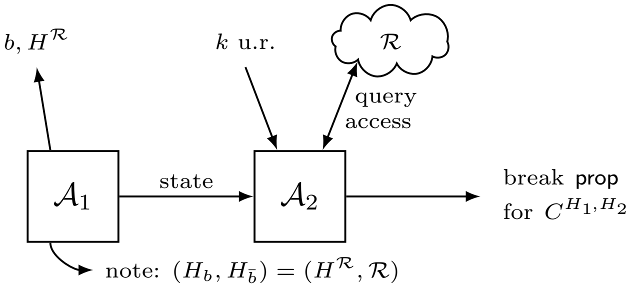

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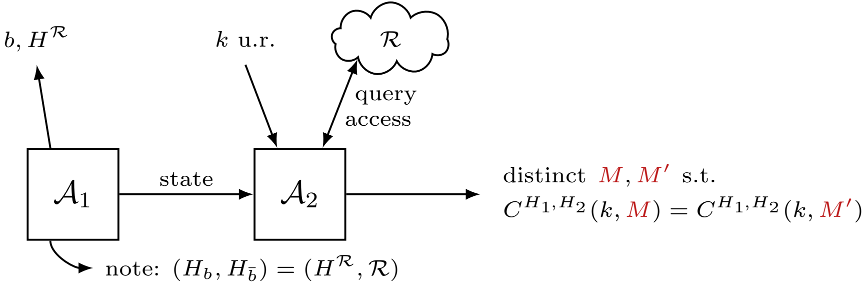

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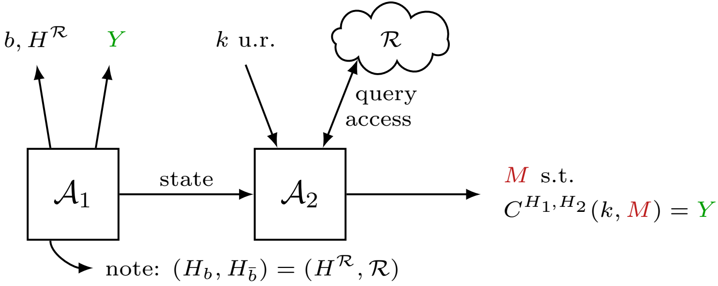

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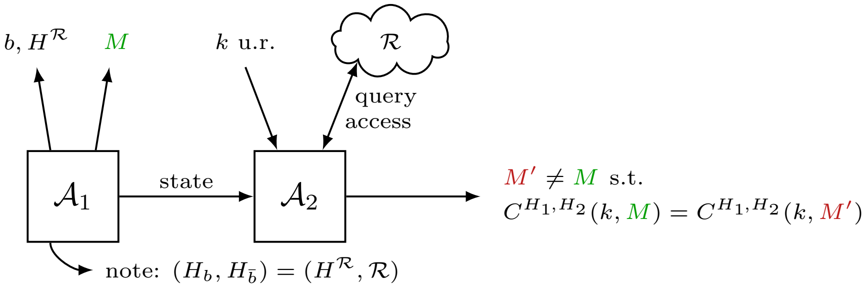

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C_{xor} is insecure

- If $\mathcal{A}_1$ chooses $H^{\mathcal{R}} = \mathcal{R}$, we have $C_{\text{xor}}^{H_1,H_2}(M) = 0$

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Can we build secure “short combiner”?

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Cryptophia's Short Combiner [Mittelbach-ACNS13]

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- Proven: $2^{n/2}$ collision security
  2^n (second) preimage security

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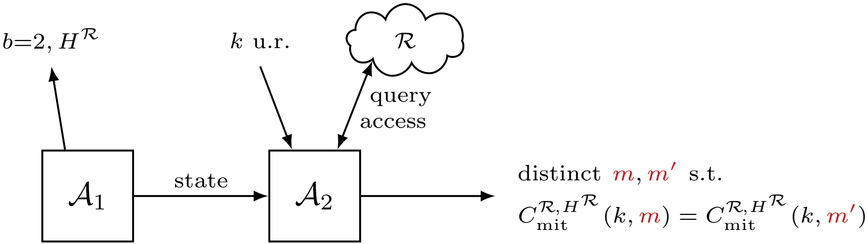

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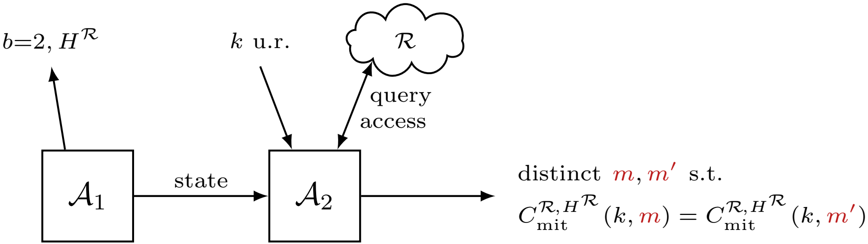

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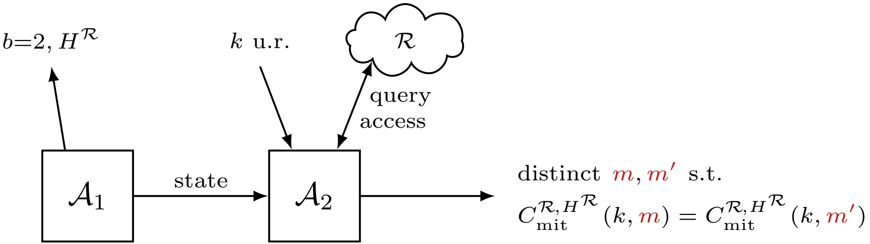

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- Generalizes to second preimage resistance (where $\mathcal{A}_1$ chooses m)

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2. Keys k_3, k_4, k_5, k_6 have become redundant

New Short Combiner

$$C^{H_1,H_2}(kl, M) = H_1\left(\tilde{m}_1^1 \| \cdots \| \tilde{m}_\ell^1\right) \oplus H_2\left(\tilde{m}_1^2 \| \cdots \| \tilde{m}_\ell^2\right)$$

$kl = (k_1, k_2, l_1, l_2)$ is a fixed key

$$m_1 \| \cdots \| m_\ell = M \| \mathsf{pad}(M)$$

$$\tilde{m}_j^1 = H_1(0 \| l_1 \| m_j \oplus k_1) \oplus H_2(0 \| l_2 \| m_j \oplus k_2) \quad (\forall j)$$

$$\tilde{m}_j^2 = H_1(1 \| l_1 \| m_j \oplus k_1) \oplus H_2(1 \| l_2 \| m_j \oplus k_2) \quad (\forall j)$$

Changes

1. Use extra keys l_1, l_2 to impose injectivity (w.h.p.)
2. Keys k_3, k_4, k_5, k_6 have become redundant
3. Simplifications in notation

Security

Theorem

- Adversary $\mathcal{A}$ makes at most $q_{\mathcal{A}}$ queries to C^{H_1,H_2}
- $H^{\mathcal{R}}$ makes at most q_H calls to $\mathcal{R}$

$$\mathbf{Adv}^{\mathsf{coll}}(\mathcal{A}) \leq 2q_H^3 q_{\mathcal{A}}^2/2^n$$
$$\mathbf{Adv}^{\mathsf{sec}}(\mathcal{A}) \leq 4q_H^3 q_{\mathcal{A}}/2^n$$
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- One can assume $q_H = \mathcal{O}(1)$

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- Tighter bounds in paper

Proof Idea

$$\tilde{m}^1(kl, m) = H_1(0 \parallel l_1 \parallel m \oplus k_1) \oplus H_2(0 \parallel l_2 \parallel m \oplus k_2)$$
$$\tilde{m}^2(kl, m) = H_1(1 \parallel l_1 \parallel m \oplus k_1) \oplus H_2(1 \parallel l_2 \parallel m \oplus k_2)$$

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Lemma

- For any kl and any m, m':

  $$\tilde{m}^1(kl, m) \quad \tilde{m}^2(kl, m) \quad \tilde{m}^1(kl, m') \quad \tilde{m}^2(kl, m')$$

  are "more or less" mutually unrelated

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Consequences

- Preprocessing functions injective (w.h.p.)
- $H^{\mathcal{R}}$-evaluation "cancels out" an $\mathcal{R}$-call w.p. $\leq q_H^3/2^n$

Conclusions

Our Results

- Constant time attacks on Cryptophia's short combiner
- Fix to re-establish security claims

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- Different security properties?
- Less H_1/H_2-calls?
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Thank you for your attention!