On the Indifferentiability of the Grøstl Hash Function

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Joint work with: Elena Andreeva and Bart Preneel

Security and Cryptography for Networks
Amalfi, Italy

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Outline

1. Preliminaries
2. Differentiability of Grøstail
3. Indifferentiability of Grøstl
4. Conclusions

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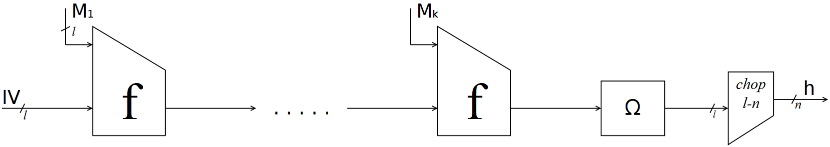
2 Differentiability of Grøstail

3 Indifferentiability of Grøstl

4 Conclusions

Grøstl



- Grøstl: a second round SHA-3 candidate
- Grøstl supports digests of length $n = 224, 256, 384$ or 512 bits
- State size l larger than output size n: $l \geq 2n$

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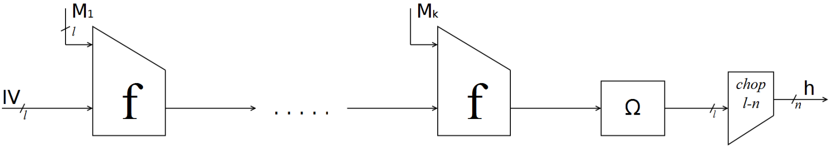

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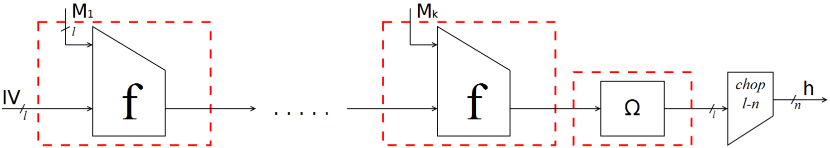

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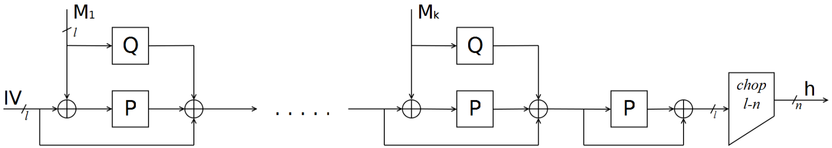

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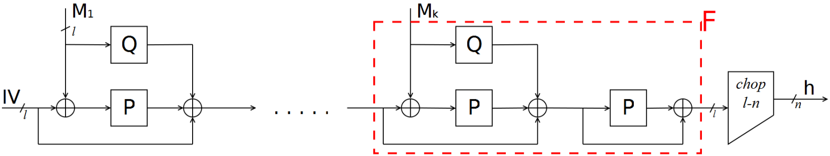

- Grøstail: last compression function with the final transformation (i.e., Grøstail is the 'tail' of Grøstl)

$$\mathrm{F}(h, m) = P(f(h, m)) \oplus f(h, m)$$

Indifferentiability

- Distinguisher $\mathcal{D}$ tries to differentiate a function C from a random oracle RO. $\mathcal{D}$ is allowed to know the underlying structure of C

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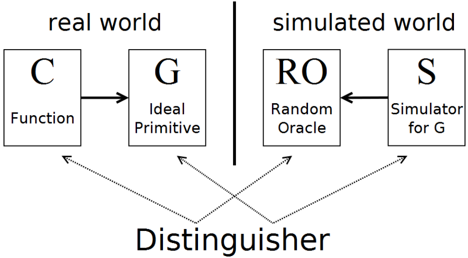

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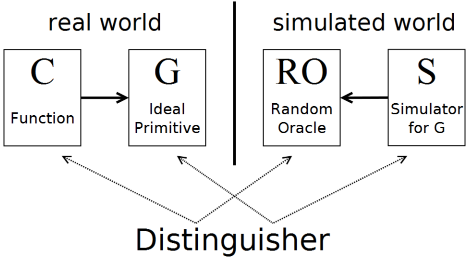
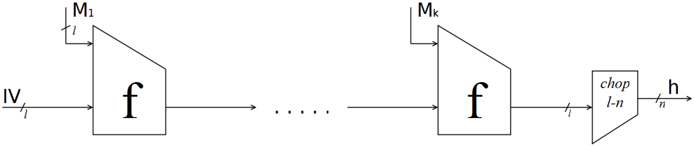


- C with oracle access to G is indifferentiable from RO if there exists a simulator S such that for any $\mathcal{D}$ these games are indistinguishable
- Throughout: G denotes (P, Q), and C is either Grøstl or Grøstail

Assumption: P, Q are independent random permutations

Examples of Indifferentiable Constructions

- For f a random function (chop-Merkle-Damgård) [CDMP05]



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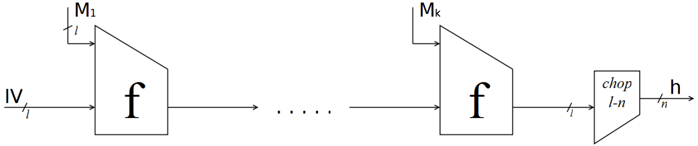

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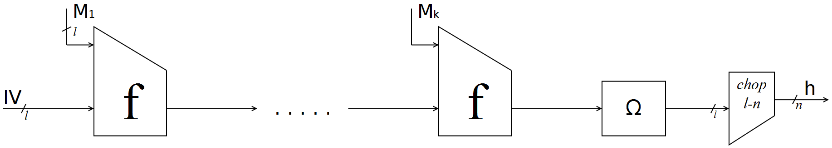

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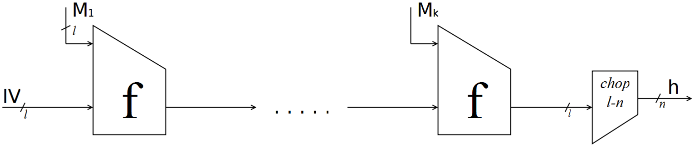

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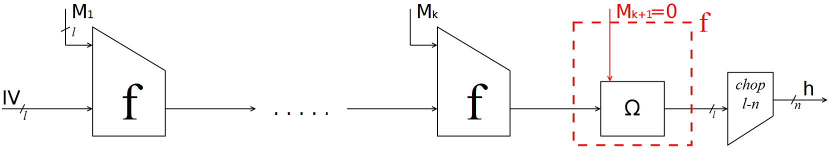

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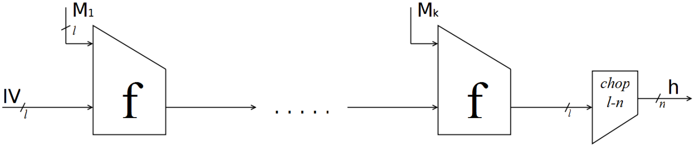

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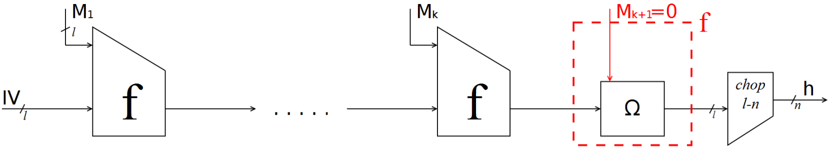

- However, for Grøstl, f is non-random (fixed-points $f(h, m) = h$)

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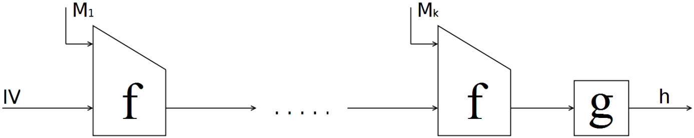

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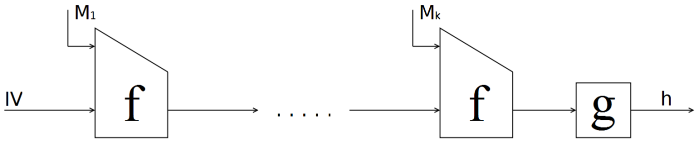

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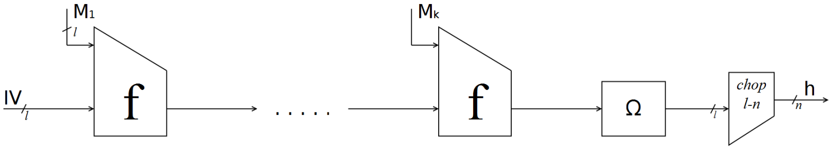

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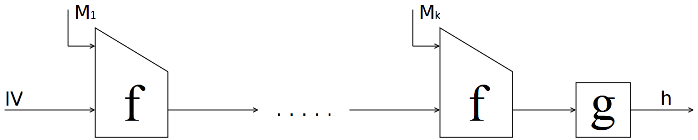

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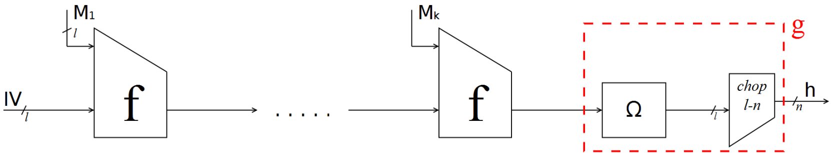

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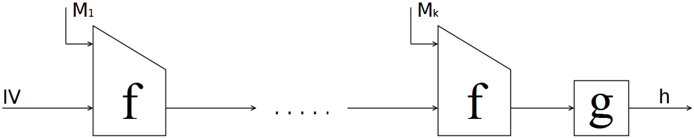

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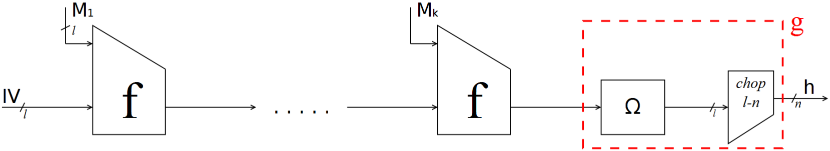

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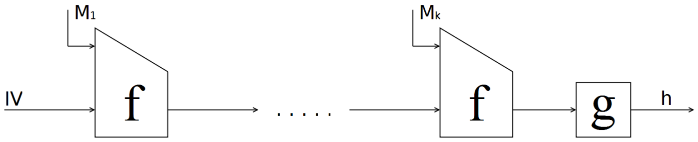

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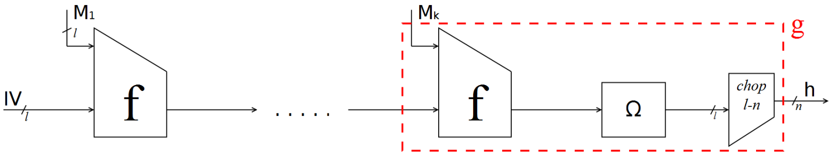

- However, for Grøstl, Ω is non-random (fixed-points $\Omega(h) = h$)
- Second attempt: consider $g := \mathrm{chop} \circ \Omega \circ f = \mathrm{chop} \circ F$

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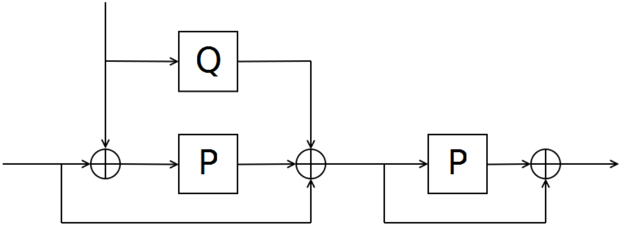

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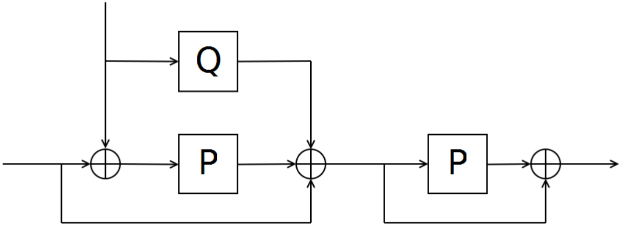

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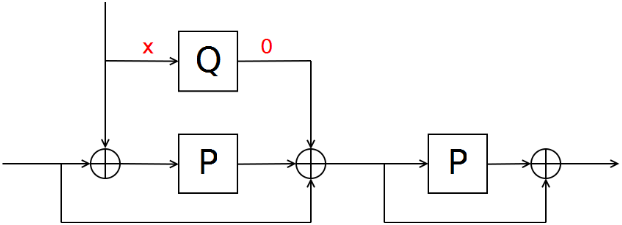

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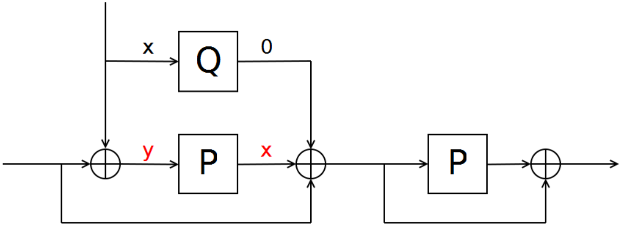

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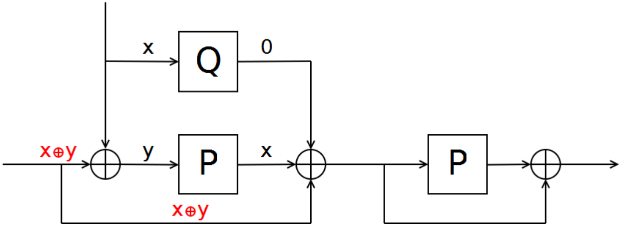

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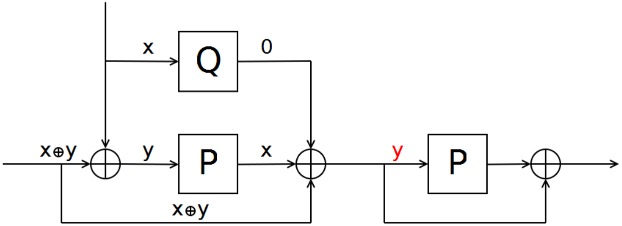

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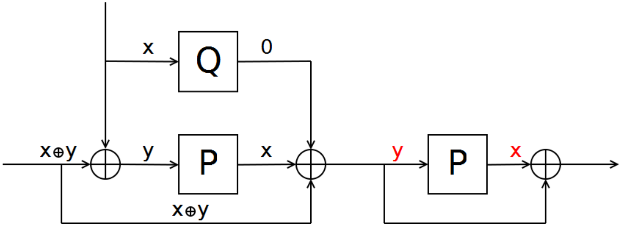

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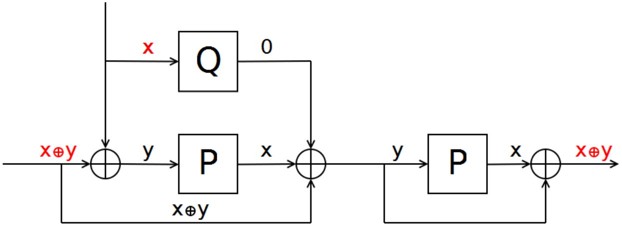

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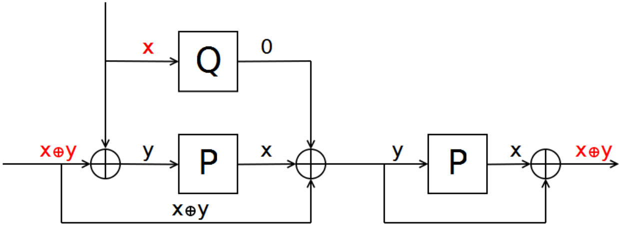

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  - For Grøstail, we have $F(x \oplus y, x) = x \oplus y$
- The simulator answers queries to Q^{-1} and P^{-1} such that the same relation holds for the random oracle: $RO(x \oplus y, x) = x \oplus y$
  $\rightarrow$ fixed-point for RO

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3 Indifferentiability of Grøstl
- How to Prove Indifferentiability?
- Design of the Simulator

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How to Prove Indifferentiability?

- Construct a simulator S that mimics behavior of P, Q
- Answers from simulator need to be consistent with RO:

  any relation between Grøstl and P, Q, also holds between RO and S

- Simulator has access to RO

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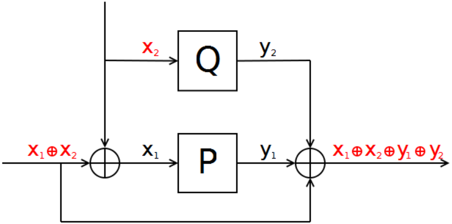
- Suppose $P(x_1) = y_1$ and $Q(x_2) = y_2$
  Then $f(x_1 \oplus x_2, x_2) = x_1 \oplus x_2 \oplus y_1 \oplus y_2$



- Edge denoted by $x_1 \oplus x_2 \xrightarrow{x_2} x_1 \oplus x_2 \oplus y_1 \oplus y_2$

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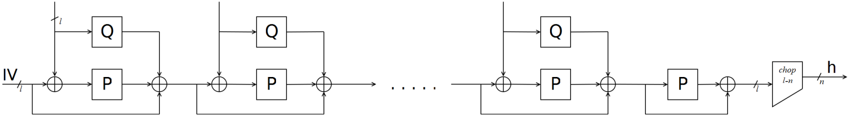

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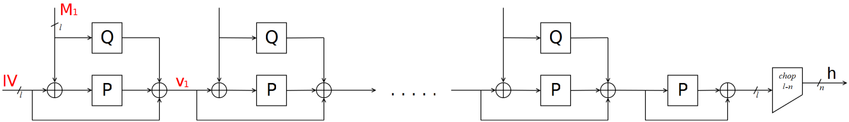

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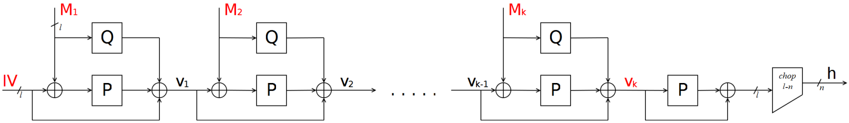

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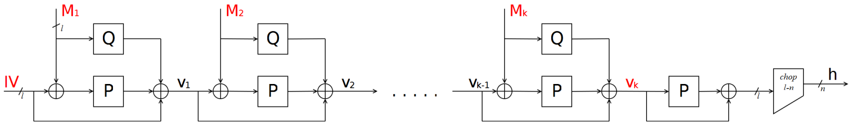

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- Suppose $(M_1, \dots, M_k) = \mathrm{pad}(M)$ for some M
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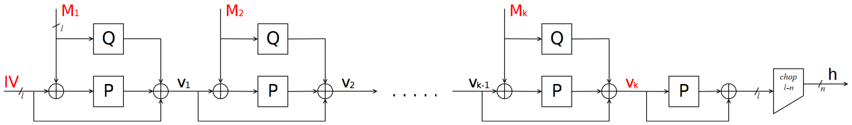

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Simulator can only guarantee this if v_k is queried to P *after* path to v_k is paved!

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- Simulator mimics $P, Q \rightarrow$ four interfaces $\mathsf{S}_P, \mathsf{S}_Q, \mathsf{S}_{P^{-1}}, \mathsf{S}_{Q^{-1}}$
- Simulator increases $\bar{r}(V) \cap \mathrm{dom}(P)$ and $r(V)$ as little as possible
  - $\bar{r}(V) \cap \mathrm{dom}(P)$: if $IV \xrightarrow{M} v$, we require $\mathrm{chop}_{l-n}(P(v) \oplus v) = RO(M)$
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- Sometimes, simulator may be forced to increase these sets

	S_P	S_Q	$\mathsf{S}_{P^{-1}}$	$\mathsf{S}_{Q^{-1}}$
$\bar{r}(V) \cap \mathrm{dom}(P)$	assure $\mathrm{chop}(P(v) \oplus v) = RO(M)$	—	—	—
$r(V)$	assure min. increase	assure min. increase	—	—

On query $S_P(x_1) \to y_1$:

$y_1 \xleftarrow{\$} \mathbb{Z}_2^l \backslash \mathrm{rng}(P)$
if S is forced to increase $r(V)$:
    assure minimal increase
if S is forced to increase $\bar{r}(V) \cap \mathrm{dom}(P)$:
    assure $\mathrm{chop}_{l-n}(y_1 \oplus x_1) = RO(M)$

On query $S_Q(x_2) \to y_2$:

$y_2 \xleftarrow{\$} \mathbb{Z}_2^l \backslash \mathrm{rng}(Q)$
if S is forced to increase $r(V)$:
    assure minimal increase
avoid increasing $\bar{r}(V) \cap \mathrm{dom}(P)$

On query $S_{P-1}(y_1) \to x_1$:

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avoid increasing $r(V)$
avoid increasing $\bar{r}(V) \cap \mathrm{dom}(P)$

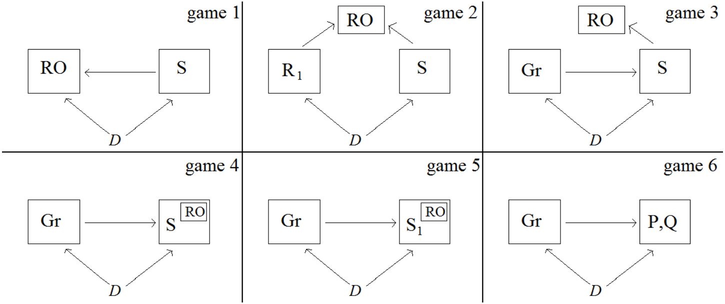
On query $S_{Q-1}(y_2) \to x_2$:

$x_2 \xleftarrow{\$} \mathbb{Z}_2^l \backslash \mathrm{dom}(Q)$
avoid increasing $r(V)$

- In every 'assure' and 'avoid', there is a **GOTO**-statement

	S_P	S_Q	S_{P-1}	S_{Q-1}
$\bar{r}(V) \cap \mathrm{dom}(P)$	assure $\mathrm{chop}(P(v) \oplus v) = RO(M)$	—	—	—
$r(V)$	assure min. increase	assure min. increase	—	—

Proof (idea)



- R_1 is a relay algorithm
- S_1 is S with **GOTO**-statements removed

$$\mathrm{Adv}(\mathcal{D}) = O(\Pr(\textbf{GOTO}\text{ is executed})) = O\left(\frac{(Kq)^4}{2^l}\right)$$

($\mathcal{D}$ makes at most q queries of length at most K message blocks)

Our Result

$$\mathrm{Adv}(\mathcal{D}) = O\left(\frac{(Kq)^4}{2^l}\right)$$

($\mathcal{D}$ makes at most q queries of length at most K message blocks)

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- Other second round SHA-3 candidates, like JH, Keccak and Shabal, have recently also been proven indifferentiable

	bound for $n = 256$
Grøstl	$O(q^4/2^{512})$
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- Would Grøstl have state size 1024, we would have $O(q^4/2^{1024})$

Outline

1. Preliminaries
2. Differentiability of Grøstail
3. Indifferentiability of Grøstl
4. Conclusions

Conclusions

- We proved the indifferentiability of Grøstl, under the assumption that P, Q are two independent random permutations
- Grøstl behaves like a random oracle up to $2^{n/2}$ queries

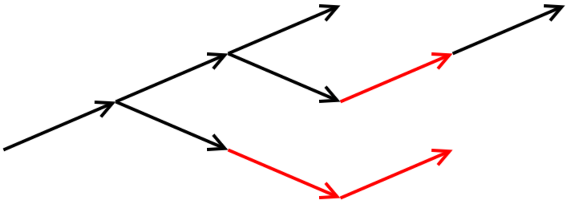
- Existing results could not be carried over to Grøstl:
  - Compression function and final transformation are differentiable
  - We additionally demonstrated that Grøstail is differentiable

- Open problem: improving the bound

Thank you for your attention!

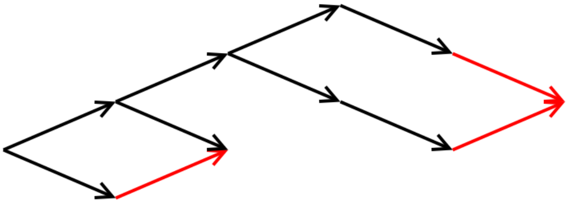
SUPPORTING SLIDES!!!

Design of the Simulator



- Black = old edges, Red = newly added edges in a certain query
- When adding edges to tree, make sure that:
  1. End node of edge has no outgoing edge in the updated graph

Design of the Simulator



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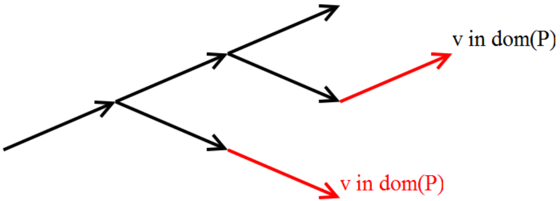

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- When adding edges to tree, make sure that:
  1. End node of edge has no outgoing edge in the updated graph
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  3. $\bar{r}(V)$ is *never* increased with a node in the updated $\mathrm{dom}(P)$

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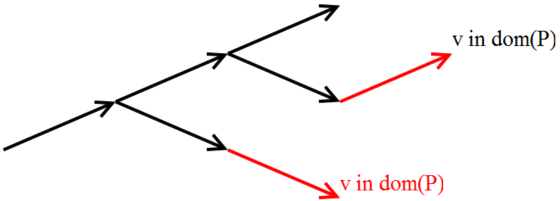

- Black = old edges, Red = newly added edges in a certain query
- When adding edges to tree, make sure that:
  1. End node of edge has no outgoing edge in the updated graph
  2. End node of edge has no incoming edge in the updated graph (except for the edge itself, of course)
  3. $\bar{r}(V)$ is *never* increased with a node in the updated $\mathrm{dom}(P)$
- In general: $r(V)$ and $\bar{r}(V) \cap \mathrm{dom}(P)$ should be increased as little as possible

Design of the Simulator

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  - Simulator is forced to increase $r(V)$
    - Make sure that 1.-3. (of previous slide) are satisfied
  - Simulator is forced to increase $\bar{r}(V) \cap \mathrm{dom}(P)$
    - This only happens if $\mathcal{D}$ queries $P(v)$ for $v \in \bar{r}(V)$
    - Simulator uses RO to output $P(v)$ such that $\mathrm{chop}_{l-n}(P(v) \oplus v) = RO(M)$
- Notice that $\bar{r}(V)$ only increases if $r(V)$ increases

Proof (idea)

$$\Pr\left(\mathcal{D}^{RO,\mathsf{S}^{RO}}=1\right)=\Pr\left(\mathcal{D}^{G_1}=1\right)$$

$$\Pr\left(\mathcal{D}^{G_1}=1\right)=\Pr\left(\mathcal{D}^{G_2}=1\right)$$

$$\left|\Pr\left(\mathcal{D}^{G_2}=1\right)-\Pr\left(\mathcal{D}^{G_3}=1\right)\right|\leq\Pr\left(\mathcal{D}^{G_2}\text{ sets }\textbf{bad}\right)+\Pr\left(\mathcal{D}^{G_3}\text{ sets }\textbf{bad}\right)$$

$$\Pr\left(\mathcal{D}^{G_3}=1\right)=\Pr\left(\mathcal{D}^{G_4}=1\right)$$

$$\left|\Pr\left(\mathcal{D}^{G_4}=1\right)-\Pr\left(\mathcal{D}^{G_5}=1\right)\right|\leq\Pr\left(\mathcal{D}^{G_4}\text{ sets }\textbf{bad}\right)$$

$$\left|\Pr\left(\mathcal{D}^{G_5}=1\right)-\Pr\left(\mathcal{D}^{G_6}=1\right)\right|\leq\frac{r_P^2}{2^l}$$

$$\Pr\left(\mathcal{D}^{G_6}=1\right)=\Pr\left(\mathcal{D}^{\mathsf{Gr}^{P,Q},(P,Q)}=1\right)$$

$$\begin{aligned}\left|\Pr\left(\mathcal{D}^{\mathsf{Gr}^{P,Q},(P,Q)}=1\right)-\Pr\left(\mathcal{D}^{RO,\mathsf{S}^{RO}}=1\right)\right|&\leq 3\Pr\left(\mathcal{D}^{G_4}\text{ sets }\textbf{bad}\right)+\frac{r_P^2}{2^l}\\&=O\left(\frac{(Kq)^4}{2^l}\right)\end{aligned}$$