Formal Reasoning 2016
Test Block 3: Languages & Automata
(19/10/16)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question you can score ten points. Good luck!

1. We define the language

    $$L_1 := \{w \in \{a, b\}^* \mid \text{no two } a\text{'s in } w \text{ are next to each other}\}$$

    (a) Give a regular expression for this language.

    (b) Give a finite automaton that recognizes this language.

    (c) Give a context-free grammar for this language which uses only one nonterminal, namely S.

    Note: Grammars that produce language L_1 using more than one nonterminal will also score some points.

2. We define the grammar G_2 by the rules:

    $$S \to aA \mid aB$$
    $$A \to aS$$
    $$B \to BB \mid b \mid \lambda$$

    (a) Write G_2 as a triple $\langle \Sigma, V, R \rangle$.

    (b) We want to show that $b \notin \mathcal{L}(G_2)$, and consider for this the property:

    $$P(w) := w \text{ does not start with } b$$

    Show that this is not an invariant of G_2.

    (c) Is G_2 right-linear? Explain your answer.

    (d) Is $\mathcal{L}(G_2)$ regular? Explain your answer.

    (e) Give a minimal finite automaton M_2 such that $L(M_2) = \mathcal{L}(G_2)$.

    Note: You don't have to prove that your M_2 has indeed a minimal number of states.

3. Does there exist a language L with alphabet $\Sigma = \{a, b\}$ such that

    $$L^* \cap \overline{L}^* = \Sigma^*$$

    holds? Explain your answer.