Formal Reasoning 2016
Test Block 6: Additional Test
(11/01/17)

Before you read on, write your name, student number and study on the answer sheet!

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question you can score fifteen points. Good luck!

1. Give a formula f_1 of propositional logic such that:

$$\neg a \wedge \neg c \vDash f_1$$
$$\neg b \wedge c \vDash f_1$$
$$f_1 \vDash \neg a \vee c$$
$$f_1 \vDash \neg b \vee \neg c$$

   Explain your answer using a truth table.

2. Use the dictionary:

V	vertices of a graph
$E(x,y)$	there is an edge between x and y

   Give a formula f_2 of predicate logic with equality that formalizes the English sentence:

   The graph is non-empty and each vertex in the graph has degree two.

3. Explain why a finite tree cannot be a model in which the formula f_2 from the previous exercise is true.

4. In this exercise we consider modal logic with Kripke semantics. We want to know whether two properties hold for all Kripke models $\mathcal{M}$ and all formulas f.

   (a) Does $\mathcal{M} \vDash f$ imply $\mathcal{M} \vDash \Box f$? Explain your answer.

   (b) Does $\mathcal{M} \vDash f \rightarrow \Box f$ hold? Explain your answer.

5. We define the language L_5 as

   $$L_5 := \{w \in \{a,b\}^* \mid w \text{ contains the substring } aba\}$$

   Give a regular expression r_5 such that

   $$\mathcal{L}(r_5) = \overline{L_5}$$

   Explain your answer.