Formal Reasoning 2017
Test Block 1: Propositional and Predicate Logic
(25/09/17)

Before you read on, write your name, student number and study on the answer sheet!

We will only look at scratch paper if it has your name on it and you refer to it on the answer sheet. If not, we prefer that you do not hand in your scratch paper.

The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

1.
$$(\neg a \vee b) \wedge ((\neg b) \vee a) \leftrightarrow a \leftrightarrow b$$

   (a) Write this formula according to the official grammar from the course notes.   (10 points)

   (b) Give the full truth table of this formula.   (10 points)

2.   *In autumn the sun is over the southern hemisphere, and therefore it is cold here now.*

Formalize this English sentence as a formula of propositional logic using the following dictionary as well as possible: (20 points)

> A it is autumn
> C it is cold
> S the sun is over the southern hemisphere

3. The following statement holds:
$$\neg(f \leftrightarrow g) \equiv (f \vee g) \wedge \neg(f \wedge g)$$

Explain what this statement *says* in terms of truth tables or models. Note that you do not have to *show* that this statement holds.   (10 points)

4.   *There is a tall woman that all nice men like.*

Formalize this English sentence as a formula of predicate logic using the following dictionary as well as possible: (20 points)

> M domain of men
> W domain of women
> $N(x)$ x is nice
> $T(x)$ x is tall
> $L(x,y)$ x likes y

5.
$$\exists x \in W \, \neg T(x) \vDash \exists x \in W \, (T(x) \rightarrow N(x))$$

Does this statement hold? Explain your answer.   (10 points)

6. Give an interpretation I_6 in the model $M_6 = (\mathbb{N}, +, 0, 1, <, \leq)$ under which the following formula is true: (10 points)

$$\exists x, y \in D \, (P(x) \wedge P(y) \wedge \forall z \in D \, [P(z) \rightarrow \neg(x = z \leftrightarrow y = z)])$$

Note that you do not need to explain your answer.