Formal Reasoning 2017
Test Block 2: Languages & Automata
(25/10/17)

Before you read on, write your name, student number and study on the answer sheet!

We will only look at scratch paper if it has your name on it and you refer to it on the answer sheet. If not, we prefer that you do not hand in your scratch paper.

The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question you can score ten points. Good luck!

1. We define a context-free grammar G_1:

$$S \to bA$$
$$A \to aA \mid bS \mid \lambda$$

We call the language produced by this grammar L_1:

$$L_1 := \mathcal{L}(G_1)$$

   (a) Give a deterministic finite automaton M_1 with $L(M_1) = L_1$.
   (b) Give a regular expression r_1 with $\mathcal{L}(r_1) = L_1$.
   (c) Is the context-free grammar G_1 right-linear? Explain your answer.
   (d) We want to show that $bab \notin \mathcal{L}(G_1)$. For this someone proposes the following property as an invariant:

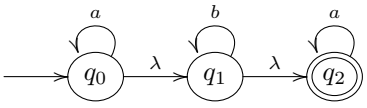
$$P(w) := \begin{array}{l} w \text{ starts with a symbol from the set } \{b, S\} \textit{ and} \\ w \text{ contains an odd number of symbols from } \{b, S\} \end{array}$$

   Does this work? Explain your answer.
   (e) Does the following equality hold?

$$L_1 = \{w \in \{a, b\}^* \mid P(w) \text{ holds}\}$$

   Explain your answer.

2. We define a non-deterministic finite automaton M_2:



We call the language recognized by this automaton L_2:

$$L_2 := L(M_2)$$

   (a) Write M_2 as a quintuple $\langle \Sigma, Q, q_0, F, \delta \rangle$. Define δ by giving equations of the form $\delta(q_i, x) = \ldots$ for all possible inputs q_i and x.
   (b) Give a regular expression r_2 with $\mathcal{L}(r_2) = L_2$.
   (c) Give a *deterministic* finite automaton M_2' with $L(M_2') = L_2$.

3. If for a language L is given that $\lambda \in L$ and $LL = L$, does it always hold that $L^* = L$?

   If so, explain why. If not, give an example of a language L_3 for which this does not hold, and explain why it is a counterexample.