

Formal Reasoning 2019
Test Block 1: Propositional and Predicate Logic
(23/09/19)

Before you read on, write your name, student number and study on the answer sheet!

We will only look at scratch paper if it has your name on it and you refer to it on the answer sheet. If not, we prefer that you do not hand in your scratch paper.

The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

1. Consider the propositional formula:

$$\neg a \vee b \leftrightarrow \neg b \vee a$$

- (a) Give the form of this formula according to the formal grammar from the course notes. (10 points)
- (b) Give the truth table of this formula. (10 points)

2. We use the dictionary:

R	it rains
W	I get wet

- (a) Give a propositional formula f_1 corresponding to the English sentence: *I get wet if it rains.* (5 points)
- (b) Give a propositional formula f_2 corresponding to the English sentence: *I only get wet if it rains.* (5 points)
- (c) Give a propositional formula f_3 corresponding to the English sentence: *I get wet, if and only if it rains.* (5 points)
- (d) Does $f_1 \wedge f_2 \equiv f_3$ hold? Explain your answer. (10 points)
3. A formula f is called *valid* when $\models f$, and *satisfiable* when $\not\models \neg f$. Give an example of a propositional formula that is satisfiable but not valid, and explain why it has these properties. (10 points)

4. We use the dictionary:

N	the natural numbers
z	the number zero
$S(x, y)$	y is the <i>successor</i> of x , that is, $y = x + 1$

- (a) Give a formula of predicate logic with equality that gives the meaning of the English sentence: *zero is not a successor of a natural number.* (10 points)
- (b) Give a formula of predicate logic with equality that gives the meaning of the English sentence: *each natural number has exactly one successor.* (15 points)
5. Does the following statement hold? (10 points)

$$\models \forall x, y \in D (x = y \rightarrow P(x) \rightarrow P(y))$$

Explain your answer.