Formal Reasoning 2021
Exam
(20/01/22)

There are six sections, with each three multiple choice questions and one open question. Each multiple choice question is worth 3 points, and the open questions are worth 6 points. The first ten points are free. Good luck!

Propositional logic: multiple choice questions

1. Which of the following formulas of propositional logic does not correctly formalize the meaning of the following English sentence

   It rains or it snows, but not both.

   The dictionary we use for this is

R	it rains
S	it snows

   (a) $(R \lor S) \land (\neg R \lor \neg S)$
   (b) $R \land \neg S \lor \neg R \land S$
   (c) $\neg(R \leftrightarrow S)$
   (d) $R \lor S \land \neg(R \land S)$

2. Is the following a formula of propositional logic?

   $$(a \to a) \to a$$

   (a) Yes, and it does have the brackets according to the official grammar from the course notes.
   (b) Yes, but it does not have the brackets according to the official grammar from the course notes.
   (c) No, a formula has to have the atom b as well.
   (d) No, implication in propositional logic is right-associative.

3. Which formula should be put on the dots to get a true statement?

   $$a \to b \equiv \ldots$$

   (a) $\neg(\neg a \lor b)$
   (b) $\neg(a \lor \neg b)$
   (c) $\neg(\neg a \land b)$
   (d) $\neg(a \land \neg b)$

Propositional logic: open question

4. The truth table of a formula that contains the atomic propositions a, b and c has eight rows.

   Give a formula with atomic propositions a, b and c, of which the truth table has three ones and five zeroes, and give three different models v_1, v_2 and v_3 that correspond to these three ones.

Predicate logic: multiple choice questions

5. Consider the English sentence

   All swans are white.

   Using the dictionary

A	domain of animals
$S(x)$	x is a swan
$W(x)$	x is white

   this can be formalized by the formula

$$\forall x \in A\,(S(x) \to W(x))$$

   Note that the sentence given above means the same as

   There are no non-white swans.

   Which formula formalizes this second statement?

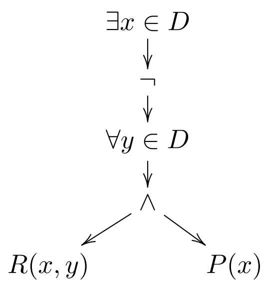
   (a) $\neg\exists x \in A\,(S(x) \to \neg W(x))$
   (b) $\neg\exists x \in A\,(S(x) \wedge \neg W(x))$
   (c) $\neg\exists x \in A\,(\neg S(x) \to \neg W(x))$
   (d) $\neg\exists x \in A\,(\neg S(x) \wedge \neg W(x))$

6. Which of the following trees correctly represents the structure of the following predicate logic formula:

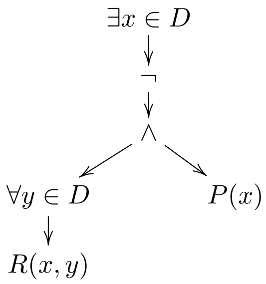
$$\exists x \in D\,(\neg\,\forall y \in D\, R(x,y) \wedge P(x))$$

   Hint: The correct answer depends on the relative binding strength of the universal quantifier versus the conjunction.

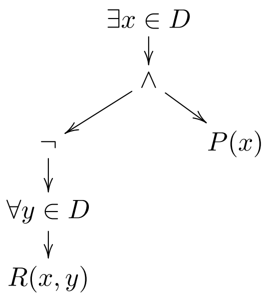
   (a)



   (b)



(c)



(d) None of the above.

7. Consider the structure

$$\langle \mathbb{N}, 0, +, \cdot \rangle$$

and the formula

$$\forall x, y \in D\,(R(x,y) \to R(y,x))$$

(this is the property of R called *symmetry*). Under which interpretation is this formula not true in this structure?

(a)

D	$\mathbb{N}$
$R(x,y)$	$x + 0 = y$

(b)

D	$\mathbb{N}$
$R(x,y)$	$x \cdot 0 = y$

(c)

D	$\mathbb{N}$
$R(x,y)$	$x + y = 0$

(d)

D	$\mathbb{N}$
$R(x,y)$	$x \cdot y = 0$

Predicate logic: open question

8. Using the dictionary

H	domain of people
s	Sharon
$L(x,y)$	x loves y

formalize the meaning of the English sentence

Only Sharon is loved by everyone.

as a formula of predicate logic with equality.

Languages: multiple choice questions

9. Is the statement

$$\lambda \in \varnothing$$

meaningful? To be more precise, can this be interpreted in such a way that it is indeed a statement within language theory that is either true or false?

   (a) Yes, and this statement is true.
   (b) Yes, but this statement is not true.
   (c) No, λ and $\varnothing$ are regular expressions.
   (d) No, $\varnothing$ does not have elements.

10. Does the following equality hold?

$$\mathcal{L}(a^*b^*) = \mathcal{L}((ab)^*)$$

   (a) Yes.
   (b) No, but $\mathcal{L}(a^*b^*) \subset \mathcal{L}((ab)^*)$.
   (c) No, but $\mathcal{L}((ab)^*) \subset \mathcal{L}(a^*b^*)$.
   (d) No, and neither language is a subset of the other.

11. Consider the context-free grammar G_{11}:

$$\begin{aligned} S &\to aA \mid \lambda \\ A &\to bS \end{aligned}$$

Someone wants to show that

$$aba \notin \mathcal{L}(G_{11})$$

using the property

$$P(w) := w \text{ has the same number of } a\text{'s and } b\text{'s}$$

in the hope that it is an invariant for this grammar. Does that work?

   (a) Yes, as the property holds for all elements in $\mathcal{L}(G_{11})$.
   (b) Yes, $S \to aA \to abS$, so in each production the number of a's and b's stays the same.
   (c) No, this is not an invariant, as the production steps $aAa \to abSa \to aba$ show.
   (d) No, this is not an invariant, as the production step $S \to aA$ shows.

Languages: open question

12. Give a right-linear context-free grammar for the language

$$L_{12} := \{w \in \{a,b\}^* \mid w \text{ does not contain } aba\}$$

Hint: It may help to make a deterministic finite automaton on scratch paper for this language first and then derive a right-linear grammar for it. You only have to hand in the grammar!

Automata: multiple choice questions

13. Let $M = \langle \Sigma, Q, q_0, F, \delta \rangle$ be a non-deterministic finite automaton. Which statement is correct for each $q_i \in Q$ and $x \in \Sigma$?

    (a) $\delta(q_i, x) \in \Sigma$
    (b) $\delta(q_i, x) \subseteq \Sigma$
    (c) $\delta(q_i, x) \in Q$
    (d) $\delta(q_i, x) \subseteq Q$

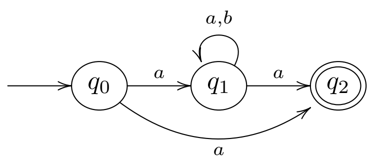
14. Let be given an arbitrary deterministic finite automaton M. We want to make another deterministic finite automaton M', such that

    $$\mathcal{L}(M') = \overline{\mathcal{L}(M)}$$

    (so M' should recognize the complement of the language that M recognizes.) How can we do this?

    (a) M' is like M, but we change all the final states to a non-final state, and all non-final states to a final state.
    (b) M' is like M, but we reverse all transitions.
    (c) M' is like M, but we add a sink state.
    (d) M' is like M, but with the sink state removed.

15. Let be given the following non-deterministic finite automaton M_{15}:

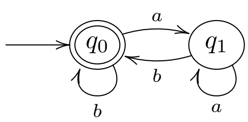


    This recognizes the words that both start and end with the symbol a. How many states has a deterministic finite automaton that recognizes the same language as M_{15} and has a minimal number of states?

    (a) Less than three states.
    (b) Exactly three states.
    (c) Exactly four states.
    (d) More than four states.

Automata: open question

16. Let be given the deterministic finite automaton M_{16}:



    List all words in $\mathcal{L}(M_{16})$ with length ≤ 3.

Discrete mathematics: multiple choice questions

17. The complete bipartite graphs $K_{m,n}$ with $m, n \geq 2$ have a Hamiltonian circuit if and only if ...

    (a) $m = n$.
    (b) m and n are both even.
    (c) m is even, n is even, or both.
    (d) None of the above.

18. We recursively define a function f with two natural number arguments, using the following recursion equations:

$$f(m, 0) = m$$
$$f(m, n+1) = m + f(m, n) \qquad \text{for all } n \geq 0$$

    Which of the following is true?

    (a) $f(m, n) = m + n$
    (b) $f(m, n) = m \cdot n$
    (c) $f(m, n) = m \cdot (n + 1)$
    (d) None of the above.

19. The sum of the numbers in any row of Pascal's triangle is ...

    (a) A power of two.
    (b) The factorial of a number.
    (c) A Bell number.
    (d) None of the above.

Discrete mathematics: open question

20. We define recursively:

$$a_0 = 4$$
$$a_{n+1} = 2a_n + 1 \qquad \text{for all } n \geq 0$$

    Prove by induction that $a_n = 5 \cdot 2^n - 1$ for all $n \geq 0$. Make sure to follow the template.

Modal logic: multiple choice questions

21. We use the dictionary

E	I exist

    What is the meaning of the following formula of doxastic logic?

$$\Box E \wedge (\Box E \rightarrow E)$$

    (a) I believe that I exist, and therefore I exist.
    (b) I know that I exist, and therefore I exist.

(c) I am obliged to exist, and therefore I exist.

(d) None of the above.

22. Are all formulas of the form

$$\Box f \to \Diamond f$$

true in all reflexive Kripke models?

(a) Yes, as this formula is true in *all* Kripke models.

(b) Yes: every world x will be its own successor, so $x \Vdash \Box f$ implies $x \Vdash \Diamond f$, as f will be true in x.

(c) No, reflexive models correspond to axiom scheme T, but this is axiom scheme D, which corresponds to serial Kripke models.

(d) No, there is a reflexive model in which $\Box f \to \Diamond f$ does not hold.

23. Consider the LTL Kripke model $\langle W, R, V \rangle$ with

$$\begin{aligned} V(x_0) &= \{a\} \\ V(x_1) &= \varnothing \\ V(x_2) &= \{b\} \\ V(x_i) &= \varnothing && \text{for all } i \geq 3 \end{aligned}$$

In which worlds x_i does the following hold?

$$x_i \Vdash (\neg a)\, \mathcal{U}\, b$$

(a) In all worlds.

(b) In worlds x_0, x_1 and x_2.

(c) In world x_1.

(d) None of the above.

Modal logic: open question

24. Give an LTL formula that expresses the requirements:

- At each time a or b is true, but not both.
- Whenever a is true, then b is true for the next two time instants, and after that a becomes true again.