Formal Reasoning 2021
Test Block 1: Propositional and Predicate Logic
(27/09/21)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

Propositional logic

1. Which of the following formulas correctly formalizes the following English sentence?

   I'm only free on Sunday

   Use for a dictionary:

F	I am free
S	it is Sunday

   (a) $F \rightarrow S$

   (b) $\neg S \rightarrow \neg F$

   (c) $\neg(F \wedge \neg S)$

   (d) all of the above

2. Which of the following statements holds?

   (a) $a \rightarrow b \rightarrow c \equiv (a \rightarrow b) \rightarrow c$

   (b) $a \rightarrow b \rightarrow c \vDash (a \rightarrow b) \rightarrow c$

   (c) $(a \rightarrow b) \rightarrow c \vDash a \rightarrow b \rightarrow c$

   (d) all of the above

3. In which model is the formula $\neg(a \rightarrow b)$ true?

   (a) $v(a) = 1 \quad v(b) = 1$

   (b) $v(a) = 0 \quad v(b) = 1$

   (c) $v(a) = 0 \quad v(b) = 0$

   (d) none of the above

4. Give a formula f that only uses the atomic propositions a and b and the connectives $\neg$ and $\wedge$, for which

   $$f \equiv a \leftrightarrow b$$

   Write your solution according to the official grammar from the course notes.

Predicate logic

5. Translate the following English sentence into a formula of predicate logic:

   Apart from humans, the only other animal to get sunburnt is the pig.

   Use for a dictionary:

A	domain of animals
$H(x)$	x is human
$P(x)$	x is a pig
$B(x)$	x is sunburnt

   Try to include all information from this sentence in your solution. For example, also represent the word 'other'. You do not need to write your solution according to the official grammar from the course notes.

6. Someone wants to formalize the sentence

   There is an intelligent man that loves Sharon.

   using the dictionary from the course notes and gives the wrong answer

   $$\exists x \in M \left(I(x) \to L(x, s) \right)$$

   accidentally using an implication instead of a conjunction. Which of the following formulas is logically equivalent to this wrong formula?

   (a) $\left(\exists x \in M \, I(x) \right) \to \left(\exists x \in M \, L(x, s) \right)$

   (b) $\left(\exists x \in M \, \neg I(x) \right) \vee \left(\exists x \in M \, L(x, s) \right)$

   (c) $\neg \left(\exists x \in M \, I(x) \right) \vee \left(\exists x \in M \, L(x, s) \right)$

   (d) none of the above

7. Consider the structure $M := (\mathbb{N}, <)$ with the interpretation I that maps:

   $$N \mapsto \mathbb{N}$$
   $$L(x, y) \mapsto x < y$$

   Which of the following holds?

   (a) $(M, I) \vDash \forall x \in N \, \exists y \in N \, L(x, y)$

   (b) $(M, I) \vDash \forall x \in N \, \exists y \in N \, L(y, x)$

   (c) $(M, I) \vDash \exists x \in N \, \forall y \in N \, L(x, y)$

   (d) $(M, I) \vDash \exists x \in N \, \forall y \in N \, L(y, x)$

8. Consider the formula

   $$\forall x \in D \left(\exists y_1 \in D \, y_1 \neq x \wedge \exists y_2 \in D \, y_2 \neq x \right)$$

   What is the number of elements that the interpretation of D in a model of this formula can contain?

   (a) two or more

   (b) three or more

   (c) zero, two, or more

   (d) zero, three, or more