Formal Reasoning 2021
Test Block 2: Languages and Automata
(10/11/21)

There are six multiple choice questions and two open questions (questions 1 and 5). Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

Languages

1. Give a regular expression for the language:

$$L_1 := \{w \in \{a,b\}^* \mid w \text{ contains neither } aa \text{ nor } bb\}$$

This means that the words in L_1 have to alternate between a and b.

Since you have to type the expression in the Cirrus editor, please adhere to these rules:

- Type the alphabet symbols and parentheses simply as '`a`', '`b`', '`(`', and '`)`'.
- Type the regular expression $\emptyset$ as '`0`' (a zero).
- Type the regular expression λ as '`^`'.
- Type the operator $\cup$ as '`U`'.
- Type the operator * as '`*`', so without putting it in superscript.

2. Consider the context-free grammar

$$G_2: \quad S \to a \mid SbS$$

Is the language $\mathcal{L}(G_2)$ regular?

Note that in order to get partial points for a partially correct answer, you actually have to give two answers: first a simple 'Yes' or 'No' and then a more detailed answer.

(a) Yes.
(b) No.

Choose the correct explanation:

(a) Yes, because G_2 is right linear.
(b) Yes, although G_2 is not right linear, the language $\mathcal{L}(G_2)$ also has a grammar that *is* right linear.
(c) No, because G_2 is not right linear.
(d) No, although G_2 is right linear, the language $\mathcal{L}(G_2)$ also has a grammar that is *not* right linear.

3. Consider the same context-free grammar

$$G_2 : \quad S \to a \mid SbS$$

Someone proposes the following two properties as potential invariants for this grammar:

$$P_{aa}(w) := w \text{ does not contain } aa$$
$$P_{bb}(w) := w \text{ does not contain } bb$$

Is P_{aa} an invariant of G_2?

(a) Yes.

(b) No.

Is P_{bb} an invariant of G_2?

(a) Yes.

(b) No.

4. Is it the case that if $L = L'^*$ for some language L', that then $L^* = L$?

(a) Yes, because $(L'^*)^* = L'^*$.

(b) Yes, because you take L to be L'.

(c) No, because $L \neq L'$.

(d) No, because $L^* = L$ is never true.

Automata

5. Consider again the context-free grammar

$$G_2 : \quad S \to a \mid SbS$$

Give a deterministic finite automaton M_2 such that $\mathcal{L}(M_2) = \mathcal{L}(G_2)$. You are supposed to draw the automaton on the paper appendix that you should have received. Make sure to include your name and student number at the indicated place, before handing it in at the end of the test.

In addition, you have to provide some metrics within Cirrus about the automaton that you created by filling in the blanks below.

My automaton has ... state(s), of which ... are final state(s).

6. Consider the deterministic finite automaton

$$M_6 := \langle \{a, b\}, \{q_0, q_1, q_2, q_3\}, q_0, \{q_0, q_1, q_2\}, \delta_6 \rangle$$

with

$$\begin{array}{llll} \delta_6(q_0, a) = q_1 & \delta_6(q_1, a) = q_3 & \delta_6(q_2, a) = q_1 & \delta_6(q_3, a) = q_3 \\ \delta_6(q_0, b) = q_2 & \delta_6(q_1, b) = q_2 & \delta_6(q_2, b) = q_3 & \delta_6(q_3, b) = q_3 \end{array}$$

Which of the following words is *not* in $\mathcal{L}(M_6)$?

(a) λ

(b) a

(c) aa

(d) ab

7. Consider again the language

$$L_1 := \{w \in \{a, b\}^* \mid w \text{ contains neither } aa \text{ nor } bb\}$$

What is the minimal number of states in a *non*-deterministic finite automaton that accepts L_1?

(a) one

(b) two

(c) three

(d) more than three

8. For a deterministic finite automaton $M = \langle \Sigma, Q, q_0, F, \delta \rangle$, we define $M' = \langle \Sigma, Q, q_0, F \cup \{q_0\}, \delta \rangle$.

What is the relation between the languages recognized by M and M'?

(a) $\mathcal{L}(M') = \mathcal{L}(M) \cup \{\lambda\}$

(b) $\mathcal{L}(M') \subseteq \mathcal{L}(M) \cup \{\lambda\}$, and for some M equality does not hold

(c) $\mathcal{L}(M) \cup \{\lambda\} \subseteq \mathcal{L}(M')$, and for some M equality does not hold

(d) none of the above