Formal Reasoning 2022
Test Block 1: Propositional and Predicate Logic
(26/09/22)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

Propositional logic

1. How many parentheses occur in the official form of the following formula of propositional logic?

   $$a \wedge \neg b \rightarrow c \rightarrow d \vee e$$

   You should count opening and closing parentheses separately. For example, the formula $(a \rightarrow b)$ contains two parentheses.

   (a) 6

   (b) 8

   (c) 10

   (d) a count different from the three options above

2. We want to formalize the following English sentence:

   I exercise, but only when it is not Sunday.

   For the dictionary we use:

E	I exercise
S	it is Sunday

   This sentence can be read in different ways. Which of the following four formulas of propositional logic is the best formalization?

   (a) $E \rightarrow \neg S$

   (b) $E \leftrightarrow \neg S$

   (c) $\neg S \rightarrow E$

   (d) $\neg S \leftrightarrow E$

3. Which of the following logical equivalences does not hold?

   (a) $a \rightarrow \neg b \equiv \neg a \vee \neg b$

   (b) $a \rightarrow \neg b \equiv \neg(a \wedge b)$

   (c) $a \rightarrow \neg b \equiv \neg a \rightarrow b$

   (d) $a \rightarrow \neg b \equiv b \rightarrow \neg a$

4. How many models v of propositional logic are there in which the formula $a \vee b \rightarrow b \wedge a$ does not hold. Give them all. Explain your answer.

Predicate logic

5. We want to formalize the following English sentence:

   There are tall men that are intelligent.

   For the dictionary we use:

M	men
$T(x)$	x is tall
$I(x)$	x is intelligent

   Which of the following formulas of predicate logic is the best formalization of this?

   (a) $\forall x \in M\,[T(x) \to I(x)]$
   (b) $\forall x \in M\,[T(x) \wedge I(x)]$
   (c) $\exists x \in M\,[T(x) \to I(x)]$
   (d) $\exists x \in M\,[T(x) \wedge I(x)]$

6. Formalize the following English sentence as a formula of predicate logic with equality:

   Only Sharon loves Koos.

   Write the formula according to the official grammar from the course notes and use for the dictionary:

H	humans
k	Koos
s	Sharon
$L(x, y)$	x loves y

   If you don't recall the official grammar: you will already get points for a formula with the proper meaning.

7. Which of the following models is *not* suitable for showing that

   $$\exists x \in D\,\forall y \in D\; R(x, y) \not\equiv \forall y \in D\,\exists x \in D\; R(x, y)$$

   (a) $(\mathbb{N}, =)$, where D is interpreted as $\mathbb{N}$ and $R(x, y)$ as $x = y$
   (b) $(\mathbb{N}, <)$, where D is interpreted as $\mathbb{N}$ and $R(x, y)$ as $x < y$
   (c) $(\mathbb{Z}, =)$, where D is interpreted as $\mathbb{Z}$ and $R(x, y)$ as $x = y$
   (d) $(\mathbb{Z}, <)$, where D is interpreted as $\mathbb{Z}$ and $R(x, y)$ as $x < y$

8. Does the following logical equivalence hold?

   $$\forall x \in M\,[\forall y \in M\; T(y) \to T(x)] \equiv \forall x \in M\,[\forall y \in M[T(y) \to T(x)]]$$

   The difference between the two formulas being compared here, is that the right formula has additional brackets.

   (a) Yes, those two formulas are the same.

(b) Yes, these formulas are logically equivalent. The formulas differ, but they both mean that if a man y is tall, then a man x is also tall.

(c) No. In the model $(\text{men}, \text{tall})$ where M is interpreted as 'men' and $T(x)$ as 'x is tall', the left formula holds but the right formula does not hold, because some men are tall and some men aren't.

(d) No. In the model $(\text{men}, \text{tall})$ where M is interpreted as 'men' and $T(x)$ as 'x is tall', the left formula does not hold but the right formula does hold, because some men are tall and some men aren't.