Formal Reasoning 2022
Test Block 2: Languages and Automata
(10/11/22)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

Languages

1. Does the equality
$$(LL)^* = L^*L^*$$
hold for every language L?
   (a) Yes, and both languages are always equal to L^*.
   (b) Yes, but they are not always equal to L^*.
   (c) No, if you take $L = \{a, b\}$ then the word ab is only in one of the two languages $(LL)^*$ and L^*L^*.
   (d) No, if you take $L = \{a, b\}$ then the word a is only in one of the two languages $(LL)^*$ and L^*L^*.

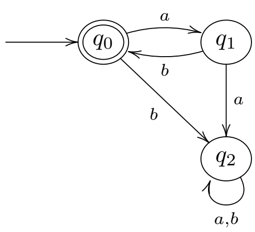
2. $$\mathcal{L}((ab)^*(ba)^*) \cap \mathcal{L}((ba)^*(ab)^*) = \ldots$$
   (a) $\mathcal{L}((ab)^* \cup (ba)^*)$
   (b) $\mathcal{L}((ab)^* \cap (ba)^*)$
   (c) $\mathcal{L}(\lambda)$
   (d) None of the above.

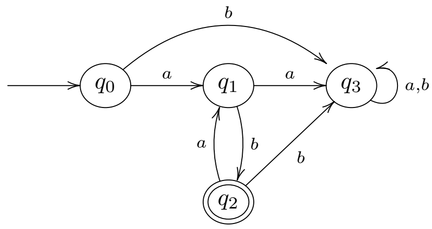
3. Is the language
$$\{a^n b^n c^m \mid n, m \in \mathbb{N}\}$$
context-free? Explain your answer.

4. Consider the context-free grammar G:
$$S \to aA$$
$$A \to aA \mid bA \mid \lambda$$
and the property
$$P(w) := \big(w \text{ starts with } a\big)$$
Is this property an invariant for this grammar?
   (a) Yes, because the property holds for all words in $\mathcal{L}(G)$.
   (b) Yes, because once a word in a production starts with a this will not change anymore.
   (c) No, the property does not hold for S.
   (d) No, $A \to aA$ is a production step, and the property holds for aA but not for A.

Automata

5. Consider the two deterministic finite automata M_1 and M_2 given by the state diagrams:





What is the relation between the two languages $\mathcal{L}(M_1)$ and $\mathcal{L}(M_2)$?

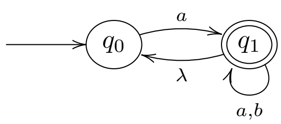
(a) $\mathcal{L}(M_1) = \mathcal{L}(M_2)$.
(b) $\mathcal{L}(M_1) \subset \mathcal{L}(M_2)$.
(c) $\mathcal{L}(M_2) \subset \mathcal{L}(M_1)$.
(d) None of the above.

6. Give a right-linear context-free grammar with at most three non-terminals for the language:

$$\{w \in \{a, b\}^* \mid w \text{ does } \textit{not} \text{ contain } abb\}$$

Hint: Make a deterministic finite automaton for this language first.

7. Consider the non-deterministic finite automaton $\langle \Sigma, Q, q_0, F, \delta \rangle$ with the state diagram:



What is the value of $\delta(q_1, a)$?

(a) $\delta(q_1, a) = q_1$
(b) $\delta(q_1, a) = \{q_1\}$
(c) $\delta(q_1, a) = \{q_0, q_1\}$
(d) None of the above.

8. Let be given a deterministic finite automaton M with exactly three states, for which it holds that $aaaaabbbbb = a^5b^5 \in \mathcal{L}(M)$. Is it then always the case that also $a^nb^5 \in \mathcal{L}(M)$ for some $n > 5$?

(a) No, this is never the case.
(b) No, this holds for *some* automata with this property, but not for all.
(c) Yes, because when processing the first five a's, the automaton has to go through a loop (because there are not enough different states), which can be repeated.
(d) Yes, because any machine with this property will accept all words in $\mathcal{L}(a^*b^*)$.