Formal Reasoning 2022
Test Block 3: Discrete Mathematics and Modal Logic
(22/12/22)

There are six multiple choice questions and two open questions (questions 2 and 6). Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

Discrete Mathematics

1. We are looking for a graph with the following properties:

   - Each vertex in the graph has degree three.
   - The graph has a Hamiltonian cycle.
   - The graph is not bipartite.

   Which of the following graphs satisfies this?

   (a) The cube graph.
   (b) The Petersen graph.
   (c) The graph $K_{3,3}$.
   (d) None of the above.

2. We define a sequence recursively by:

$$a_0 = 0$$
$$a_{n+1} = a_n + 2n$$

   Prove by induction that $a_n = n(n-1)$ for all $n \geq 0$. Follow the template.

3. The equality

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots \binom{n}{n} = 0$$

   holds for every $n > 0$. For example for $n = 3$ this becomes:

$$\binom{3}{0} - \binom{3}{1} + \binom{3}{2} - \binom{3}{3} = 1 - 3 + 3 - 1 = 0$$

   How can one see that this equality is correct for all $n > 0$?

   (a) This follows from the fact that each row of Pascal's triangle is symmetric, i.e., we have $\binom{n}{k} = \binom{n}{n-k}$ for each $0 \leq k \leq n$.
   (b) This follows from the binomial theorem, by considering the expansion of $(1-1)^n$.
   (c) Both arguments work for all $n > 0$.
   (d) Neither argument works for all $n > 0$.

4. In how many ways can we separate 5 distinguishable objects in 3 non-distinguishable non-empty piles?

   (a) $\left[{5 \atop 3} \right]$
   (b) $\left\{ {5 \atop 3} \right\}$

   And what is the outcome as a natural number?

   (a) 25
   (b) 35
   (c) 65
   (d) 85

Modal Logic

5. Consider the dictionary:

R	it rains
U	I use an umbrella

   Using this dictionary, what does the strict implication

$$\Box(R \to U)$$

   mean in deontic logic?

   (a) When it rains, I know I use an umbrella.
   (b) When it rains, I believe I use an umbrella.
   (c) When it rains, I must use an umbrella.
   (d) When it rains, I always use an umbrella.

6. Give a Kripke model $\mathcal{M}$ that shows that axiom scheme B does not generally hold, i.e., that there is a modal formula f such that

$$\not\models (f \to \Box\Diamond f)$$

   Write your model $\mathcal{M}$ as a triple $\langle W, R, V\rangle$ and explicitly give W, R, V and a formula f. You don't need to explain your answer.

7. Does $\models_T f$ imply that $\models_D f$ for all modal formulas f?

   (a) Yes, because all serial models are reflexive.
   (b) Yes, because all reflexive models are serial.
   (c) No, because not all serial models are reflexive.
   (d) No, because not all reflexive models are serial.

8. We want to formalize the sentence

   I will work until I leave home.

as a formula of LTL. We take this sentence to mean that I will stop working (at least for some time) when I leave home, and we will allow for the possibility that I leave home immediately, in which case I actually won't work at all before that.

We use the dictionary:

W	I work
H	I am at home

Now consider the following two LTL formulas:

$$(W \wedge H) \,\mathcal{U}\, (\neg W \wedge \neg H)$$

$$(W \,\mathcal{U}\, \neg H) \wedge (H \,\mathcal{U}\, \neg W)$$

Which of these is (are) a good formalization of the sentence?

(a) The first.

(b) The second.

(c) Both.

(d) Neither.