Formal Reasoning 2022
Test Blocks 1, 2 and 3: Additional Test
(11/01/23)

There are six multiple choice questions and two open questions (questions 4 and 6). Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

1. Someone formalizes the sentence

   I am inside if and only if I am not outside.

   as

   $$\mathsf{In} \leftrightarrow \neg\mathsf{Out}$$

   using the dictionary

In	I am inside
Out	I am outside

   and notes the following logical equivalences:

   $$\mathsf{In} \leftrightarrow \neg\mathsf{Out} \equiv (\mathsf{In} \to \neg\mathsf{Out}) \wedge (\neg\mathsf{Out} \to \mathsf{In})$$

   $$\mathsf{In} \leftrightarrow \neg\mathsf{Out} \equiv (\mathsf{In} \to \neg\mathsf{Out}) \wedge (\neg\mathsf{In} \to \mathsf{Out})$$

   Now which part of these statements corresponds to the 'if' part of the sentence, and which to the 'only if' part?

   (a) The 'if' part corresponds to $(\mathsf{In} \to \neg\mathsf{Out})$ and the 'only if' part corresponds to $(\neg\mathsf{Out} \to \mathsf{In})$.

   (b) The 'if' part corresponds to $(\neg\mathsf{Out} \to \mathsf{In})$ and the 'only if' part corresponds to $(\mathsf{In} \to \neg\mathsf{Out})$.

   (c) The 'if' part corresponds to $(\mathsf{In} \to \neg\mathsf{Out})$ and the 'only if' part corresponds to $(\neg\mathsf{In} \to \mathsf{Out})$.

   (d) The 'if' part corresponds to $(\neg\mathsf{In} \to \mathsf{Out})$ and the 'only if' part corresponds to $(\mathsf{In} \to \neg\mathsf{Out})$.

2. Consider the following formula of predicate logic:

   $$\forall x \in D\ R(x, a) \to Q(b)$$

   Which of the following is this formula written according to the official grammar from the course notes?

   (a)
   $$((\forall x \in D\, R(x, a)) \to Q(b))$$

   (b)
   $$(\forall x \in D\, (R(x, a) \to Q(b)))$$

(c)
$$(\forall x \in D\,[R(x,a)] \rightarrow Q(b))$$

(d)
$$(\forall x \in D\,[R(x,a) \rightarrow Q(b)])$$

3. The six kinds of regular expressions are

$$\begin{gathered} a, b, c \ldots \\ r^* \\ r_1 r_2 \\ r_1 \cup r_2 \\ \lambda \\ \varnothing \end{gathered}$$

but couldn't some have been expressed in terms of the others?

(a) λ^* can be used in the place of $\varnothing$.

(b) $\varnothing^*$ can be used in the place of λ.

(c) Neither is the case.

(d) Both are the case.

4. Consider the context-free grammar G_4 given by:

$$\begin{aligned} S &\rightarrow aAa \mid Sbb \mid a \\ A &\rightarrow abaS \end{aligned}$$

Give an invariant that shows that $aabaa \notin \mathcal{L}(G_4)$. Don't explain your answer: just giving the invariant is sufficient.

5. Let M_5 be some deterministic finite automaton with

$$\mathcal{L}(M_5) = \{w \in \{a,b\}^* \mid w \text{ does not contain } aaa\}$$

Does the machine M_5 necessarily have a sink state?

(a) Yes, after you have processed three a's in a row, you will be in a sink state.

(b) Yes, every finite automaton can have a sink state.

(c) No, there can be multiple states that are not final, with transitions between them.

(d) No, this language is infinite, so from every state, it has to be possible to get to a final state that accepts the input.

6.
- Give a planar graph G_6 that has six vertices that all have degree three, and which is not bipartite. Write your answer in the form of a pair $\langle V, E\rangle$.

  You do not need to explain why your graph has these properties.
- Does this graph have a Hamiltonian cycle? *That* you have to explain.

7. Someone tries to define the factorial sequence $a_0 = 1$, $a_1 = 1$, $a_2 = 2$, $a_3 = 6$, $\ldots$, but does not get it completely right, because in the definition the multiplication is with n instead of $n+1$:

$$a_0 = 1$$
$$a_{n+1} = n \cdot a_n$$

What is the value of a_4 according to this incorrect definition?

(a) 6

(b) 24

(c) 120

(d) None of the above.

8. Consider all Kripke models $\mathcal{M}$ that have the property $\mathcal{M} \vDash a$. Is it the case that for all these models also $\mathcal{M} \vDash \Diamond a$?

(a) Yes, if something is *actually* the case (in this instance: 'a'), then it is obviously *possible* that it is the case.

(b) Yes, a will be true in all worlds of $\mathcal{M}$, so whatever world we access, a will hold there.

(c) No, this holds if and only if the model $\mathcal{M}$ is *serial* (the property that corresponds to the axiom scheme D, which is $\Box f \to \Diamond f$.)

(d) No, this holds if and only if the model $\mathcal{M}$ is *reflexive* (the property that corresponds to the axiom scheme T, one of which forms is $f \to \Diamond f$.)