**Formal Reasoning 2023**
**Test Block 1: Propositional and Predicate Logic**
**(25/09/23)**

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

## Propositional logic

1. The English sentence:

   I'm inside, but only if it rains.

   is ambiguous. Consider the following three formulas of propositional logic:

   (i) $\mathsf{In} \leftrightarrow \mathsf{R}$
   (ii) $\mathsf{In} \to \mathsf{R}$
   (iii) $\mathsf{R} \to \mathsf{In}$

   We use as dictionary:

   | | |
   |---|---|
   | In | I'm inside |
   | R | it rains |

   Which of these formulas describe possible meanings for this sentence?

   (a) (i) and (ii)
   (b) (i) and (iii)
   (c) (ii) and (iii)
   (d) all three

2. Give a formula of propositional logic that best matches the meaning of:

   If it doesn't rain, or if I'm inside, then I stay dry.

   Use as dictionary:

   | | |
   |---|---|
   | In | I'm inside |
   | R | it rains |
   | D | I'm dry |

   *Hint:* Consider an appropriate logical law.

   If you only can think of a solution that needs connectives other than implications, then you can give that answer for partial points.

3. How should we read the formula

   $$a \vee b \wedge c$$

   and how many ones are there in its column in its truth table?

(a) $(a \vee b) \wedge c$ and the number of ones in its truth table is five
(b) $(a \vee b) \wedge c$ and the number of ones in its truth table is not five
(c) $a \vee (b \wedge c)$ and the number of ones in its truth table is three
(d) $a \vee (b \wedge c)$ and the number of ones in its truth table is not three

4. Which of the following answers is correct?

(a) $a \to (b \to c) \vDash (a \to b) \to c$ and $(a \to b) \to c \vDash a \to (b \to c)$
(b) $a \to (b \to c) \vDash (a \to b) \to c$ and $(a \to b) \to c \nvDash a \to (b \to c)$
(c) $a \to (b \to c) \nvDash (a \to b) \to c$ and $(a \to b) \to c \vDash a \to (b \to c)$
(d) $a \to (b \to c) \nvDash (a \to b) \to c$ and $(a \to b) \to c \nvDash a \to (b \to c)$

## Predicate logic

5. What is the form according to the official grammar from the course notes of the following formula?

$$\exists x, y \in D\, [x \neq y]$$

(a) $\exists x \in D\, [\exists y \in D\, [\neg(x = y)]]$
(b) $(\exists x \in D\, (\exists y \in D\, \neg(x = y)))$
(c) $(\exists x \in D\, (\exists y \in D\, (\neg(x = y))))$
(d) none of the above

6. Translate as a formula of predicate logic with equality:

Every human has exactly two parents.

Use as dictionary

| | |
|---|---|
| $H$ | domain of humans |
| $P(x, y)$ | $x$ is a parent of $y$ |

7. Let $f$ be the formula $\forall x \in M\, \exists y \in W\, L(x, y)$ and let $g$ be the formula $\exists y \in W\, \forall x \in M\, L(x, y)$. Which of the following answers is correct?

(a) $f \vDash g$ and $g \vDash f$
(b) $f \vDash g$ and $g \nvDash f$
(c) $f \nvDash g$ and $g \vDash f$
(d) $f \nvDash g$ and $g \nvDash f$

8. Let $f$ be the formula $\forall x, y, z \in N\, [L(x, y) \to L(x, z) \vee L(z, y)]$. Now consider the following structure $M$ with interpretations $I_1$ and $I_2$:

$$M := (\mathbb{N}, <, \leq)$$

$I_1 :=$

| | |
|---|---|
| $N$ | $\mathbb{N}$ |
| $L(x, y)$ | $x < y$ |

$I_2 :=$

| | |
|---|---|
| $N$ | $\mathbb{N}$ |
| $L(x, y)$ | $x \leq y$ |

Which of the following is correct?

(a) $(M, I_1) \vDash f$ and $(M, I_2) \vDash f$

(b) $(M, I_1) \vDash f$ and $(M, I_2) \nvDash f$

(c) $(M, I_1) \nvDash f$ and $(M, I_2) \vDash f$

(d) $(M, I_1) \nvDash f$ and $(M, I_2) \nvDash f$