Formal Reasoning 2023
Test Block 2: Languages and Automata
(09/11/23)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. Good luck!

Languages

1. What is λ *not* used for in this course?
   (a) a symbol from the alphabet Σ
   (b) a word from Σ^*
   (c) a regular expression
   (d) a label for a transition in a non-deterministic finite automaton

2. For which regular expression r_2 do we have that

$$\mathcal{L}(r_2) = \{w \in \{a, b\}^* \mid w \text{ contains an even number of } b\text{'s}\}$$

   (a) $r_2 = a^*(ba^*ba^*)^*$
   (b) $r_2 = a^*(ba^*b)^*a^*$
   (c) $r_2 = (a \cup bb)^*$
   (d) all three of these possibilities

3. Consider the following context-free grammar G_3 for a language with alphabet $\{\mathsf{a}, \mathsf{e}, \mathsf{h}, \mathsf{l}, \mathsf{n}, \mathsf{o}, \mathsf{r}, \mathsf{s}, \mathsf{t}, \mathsf{v}, \mathsf{y}, \mathsf{J}, \mathsf{M}, \text{␣}\}$:

$$\begin{aligned} S &\to N\text{␣}V \mid N\text{␣}V\text{␣}N \\ N &\to \mathsf{John} \mid \mathsf{Mary} \\ V &\to \mathsf{loves} \mid \mathsf{hates} \end{aligned}$$

   How many different productions are there in G_3 for the word:

   John␣loves␣Mary

   (a) 1
   (b) 3
   (c) 6
   (d) none of the above

4. Consider the context-free grammar G_4:

$$\begin{aligned} S &\to AB \\ A &\to aA \mid \lambda \\ B &\to bB \mid \lambda \end{aligned}$$

   Someone wants to show that $ba \notin \mathcal{L}(G_4)$ and considers the property:

$$P_4(w) := \big[\text{in } w \text{ there is no } b \text{ before an } a\big]$$

   Is this a suitable invariant for this? Explain your answer.

Automata

5. We define the language L_5 by:

$$L_5 := \mathcal{L}(a^*bb^*) = \{a^n b^m \mid n \in \mathbb{N}, n \geq 0, m \in \mathbb{N}, m > 0\}$$

Consider the following statement:

> Each deterministic finite automaton M with $\mathcal{L}(M) = L_5$ has to have at least three states, because there has to be a final state which will be different from the initial state, and there also has to be a non-final state that is different from the initial state.

Is this correct?

(a) Yes, this is correct. Each M with $\mathcal{L}(M) = L_5$ will have a sink state (which for example will be reached after processing ba), which is a non-final state that is different from the initial state.

(b) Yes, this is correct. However, a machine M with $\mathcal{L}(M) = L_5$ does not need to have a sink state. There can be more than one non-final state different from the initial state.

(c) No, this is not correct. There is a DFA with less than three states for the language L_5.

(d) No, this is not correct. The minimal number of states needed for L_5 is indeed three, but the argumentation is not correct.

6. Consider the following statement:

> If a deterministic finite automaton M with alphabet $\{a, b\}$ has exactly three states, and there is a word $w \in \mathcal{L}(M)$ with $|w| = 3$, then the language $\mathcal{L}(M)$ will contain infinitely many different words.

Which of the following holds?

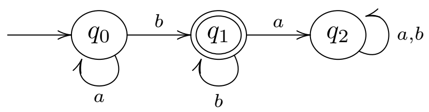
(a) This is correct. When processing the word w, at least one state will be encountered twice, and by going around that loop, one can find arbitrary large words in $\mathcal{L}(M)$.

(b) This is correct. A language of a DFA always contains infinitely many different words.

(c) This is not correct. A word w with three symbols is too short to necessarily encounter the same state twice. There is a machine with these properties that only accepts finitely many different words.

(d) This is not correct. If the automaton M has no final states, the language of M will be empty, and will certainly not contain infinitely many different words.

7. Consider the following deterministic finite automaton M_7:



Give the right linear context-free grammar that corresponds to this automaton. Do not simplify the grammar in any way, just give the result of the conversion from automata to grammars as it was described in the lectures and course notes.

8. Consider the following quintuple:

$$M_8 = \langle \{a\}, \{q_0, q_1\}, q_0, \varnothing, \delta_8 \rangle$$

with

$$\begin{aligned}
\delta_8(q_0, a) &= \varnothing \\
\delta_8(q_0, \lambda) &= \varnothing \\
\delta_8(q_1, a) &= \varnothing \\
\delta_8(q_1, \lambda) &= \{q_1\}
\end{aligned}$$

Is this a correct non-deterministic finite automaton?

(a) Yes, it is.

(b) No, the machine does not have final states.

(c) No, the machine has no transitions for the symbol a.

(d) No, the state q_1 is not reachable from the initial state q_0.