Formal Reasoning 2023
Test Blocks 1, 2 and 3: Additional Test
(11/01/24)

There are six multiple choice questions and two open questions (questions 3 and 7). Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

1. Consider the statement:
$$\neg a \vDash a \to b$$
Does this hold?
   (a) It depends on the truth values of a and b whether this holds.
   (b) Yes, this holds.
   (c) No, this does not hold. A counter example is $v(a) = 1$ and $v(b) = 0$.
   (d) No, this does not hold. A counter example is $v(a) = 1$ and $v(b) = 1$.

2. Someone tries to formalize the sentence
   There are at least three nice men.
   using the dictionary

M	the domain of men
$N(x)$	x is nice

   as the formula:
$$\exists x, y, z \in M[N(x) \wedge N(y) \wedge N(z)]$$
   Is this correct?
   (a) Yes, this is correct.
   (b) No, there also needs to be a part that states that every nice man is one of x, y or z.
   (c) No, because this formula means that there is at least *one* nice man.
   (d) No, obviously this is not a formula according to the official syntax, but it is not even a formula of predicate logic with equality.

3. Consider the context-free grammar G_3 over the alphabet $\Sigma = \{(,)\}$ given by the rules:
$$S \to (S) \mid SS \mid \lambda$$
   The language $\mathcal{L}(G_3)$ consists of exactly all words in which the parentheses '(' and ')' are properly balanced. For example, we have (()(())) $\in \mathcal{L}(G_3)$.
   Give the number of words in $\mathcal{L}(G_3)$ that have length six, and list all of them.

4. Someone claims that the following property

$$P(w) := [w \in V\Sigma^*]$$

is an invariant for every right linear context-free grammar. Is this correct?

(a) Yes, this is correct.

(b) No, the property should be:

$$P(w) := [w \in \Sigma^* V]$$

(c) No, the property should be:

$$P(w) := [w \in (V \cup \{\lambda\})\Sigma^*]$$

(d) None of the above.

5. Is there a non-deterministic finite automaton with at most two states that accepts the language $\mathcal{L}(a^*b^*)$?

(a) Yes, there even is a deterministic finite automaton with two states that accepts this language.

(b) Yes, but there is not a deterministic finite automaton with at most two states that accepts this language.

(c) No, any automaton for this language needs at least three states.

(d) No, there is no automaton for this language at all, as it is not regular.

6. Is there a graph with 15 vertices that all have degree 3?

(a) Yes, the Petersen graph is an example of such a graph.

(b) Yes, the bipartite complete graph $K_{3,15}$ is an example of such a graph.

(c) No, because at most two vertices are allowed to have an odd degree.

(d) No, because this would mean that the sum of the degrees would be odd.

7. Prove with induction that $3^n > 8$ when $n \geq 2$.

8. Someone wants to formalize the English sentence

When it rains I always get wet.

using the dictionary

R	it rains
W	I get wet

as a formula of temporal logic, and is considering the two modal formulas:

$$\Box(R \to W)$$
$$R \to \Box W$$

Which of these two is the correct formalization?

(a) The formula $\Box(R \to W)$ is correct because the formula $R \to \Box W$ would imply that once it has rained I will be wet forever, and that is not what the sentence says.

(b) The formula $R \to \Box W$ is correct, because it corresponds best to the sentence.

(c) Both formulas mean the same, so both are correct.

(d) Neither of these formulas is correct.