Formal Reasoning 2024
Test Block 1: Propositional and Predicate Logic
(23/09/24)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

Propositional logic

1. We want to formalize the English sentence

    It rains, but I don't get wet.

    For this we use the dictionary:

R	it rains
W	I get wet

    What is a correct formula of propositional logic, that corresponds to the meaning of this sentence?

    (a) $R \wedge \neg W$

    (b) $R \rightarrow \neg W$

    (c) $R \rightarrow W \vee \neg W$

    (d) None of the above.

2. What is the official form of the formula

    $$\neg a \wedge a$$

    according to the official grammar for formulas of propositional logic from the course notes?

    (a) $\neg a \wedge a$

    (b) $\neg(a \wedge a)$

    (c) $(\neg a) \wedge a$

    (d) $(\neg a \wedge a)$

3. Consider the statement

    $$(\neg f \leftrightarrow \neg g) \equiv \neg(f \leftrightarrow g)$$

    where f and g are formulas of propositional logic. What is the case?

    (a) For all formulas f and g, this holds.

    (b) There are formulas f and g for which this holds and there are formulas f and g for which this does not hold.

    (c) For all formulas f and g, this does not hold.

(d) This is not a correct statement, because the symbol $\equiv$ should not occur inside a formula of propositional logic.

4. Let f be an arbitrary proposition. Does the following hold?

If not $\vDash f$, then $\vDash \neg f$.

Explain your answer.

Predicate logic

5. Which formula of predicate logic gives the meaning of the English sentence:

   All intelligent men are nice.

   For this we use the dictionary:

M	the domain of men
$I(x)$	x is intelligent
$N(x)$	x is nice

   (a) $\neg \exists x \in M\, [I(x) \wedge \neg N(x)]$
   (b) $\neg \exists x \in M\, [I(x) \rightarrow \neg N(x)]$
   (c) $\neg \exists x \in M\, [I(x) \wedge N(x)]$
   (d) $\neg \exists x \in M\, [I(x) \rightarrow N(x)]$

6. Does the following hold?

   $$\exists y \in W\, [\forall x \in M\, L(x,y)] \vDash \forall x \in M\, [\exists y \in W\, L(x,y)]$$

   (a) Yes, because these two formulas mean the same, and therefore are logically equivalent.
   (b) Yes, but it does not hold in the other direction.
   (c) No, but it does hold in the other direction.
   (d) No, because the symbol $\vDash$ should have been the symbol $\equiv$.

7. Give an interpretation I_7 in a structure M_7 such that:

   $$(M_7, I_7) \vDash \forall x, y, z \in D\, [R(x,z) \wedge R(y,z) \rightarrow \neg R(x,y)]$$

   Use an interpretation where the domain D is interpreted as the set of natural numbers $\mathbb{N}$. Explain your answer.

8. Consider the following formula of predicate logic:

   $$\exists x \in W\, \forall y \in W\, [x = y \rightarrow L(k,y)]$$

   We use the dictionary:

W	the domain of women
k	Koos
$L(x,y)$	x loves y

   What is the meaning of this formula?

   (a) *Koos loves exactly one woman.*
   (b) *Koos loves at least one woman.*
   (c) *Koos loves at most one woman.*
   (d) None of the above.