Formal Reasoning 2024
Test Block 2: Languages and Automata
(07/11/24)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. Good luck!

Languages

1. Does the equation $(L_1L_2)^R = L_1^R L_2^R$ hold for all languages L_1 and L_2?
   (a) Yes, in both cases there is a combination of reversing and concatenating the languages.
   (b) Yes, when constructing these languages, it does not matter in which order you concatenate two words and reverse them.
   (c) No, but $(L_1L_2)^R = L_2^R L_1^R$ *does* hold, as $(uv)^R = v^R u^R$ for all words u and v.
   (d) No, $(L_1L_2)^R$ is not determined by L_1^R and L_2^R.

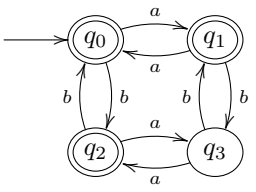
2. Which language is equal to $\overline{\mathcal{L}(a^*b^*)}$ over the alphabet $\{a, b\}$?
   (a) $\mathcal{L}\big((a \cup b)^* ba (a \cup b)^*\big)$
   (b) $\mathcal{L}(b^*a^*)$
   (c) $\mathcal{L}\big(\overline{a^*b^*}\big)$
   (d) none of the above

3. Consider the context-free grammar G_3:
   $$S \to x \mid y \mid z \mid S + S \mid SS$$
   This defines a language $\mathcal{L}(G_3)$ with alphabet $\Sigma = \{x, y, z, +\}$.
   How many parse trees does the word
   $$xy + z$$
   have?
   (a) 1
   (b) 2
   (c) 8
   (d) 16

4. Consider the context-free grammar G_4:
   $$S \to aS \mid Sb \mid \lambda$$
   and the property
   $$P(w) := \big[w \text{ does not contain } ba\big]$$
   Is this property an invariant for G_4? Explain your answer.

Automata

5. Consider the deterministic finite automaton M_5:



   We write $|w|_a$ for the number of a's in w, and likewise $|w|_b$ for the number of b's.

   Which language is accepted by M_5?

   (a) $\{w \in \{a,b\}^* \mid |w|_a + |w|_b \text{ is even}\}$
   (b) $\{w \in \{a,b\}^* \mid |w|_a + |w|_b \text{ is odd}\}$
   (c) $\{w \in \{a,b\}^* \mid |w|_a \times |w|_b \text{ is even}\}$
   (d) $\{w \in \{a,b\}^* \mid |w|_a \times |w|_b \text{ is odd}\}$

6. Consider the context-free grammar G_6:

$$S \to aS \mid Sb \mid \lambda$$

   Is there a deterministic finite automaton that accepts $\mathcal{L}(G_6)$?

   (a) Yes, because this language is regular.
   (b) Yes, all context-free languages are accepted by a deterministic finite automaton.
   (c) No, the grammar is not right linear.
   (d) No, a deterministic finite automaton for this language needs more than one state, and there is only one non-terminal in the grammar.

7. Give a deterministic finite automaton $M_7 = \langle \Sigma, Q, q_0, F, \delta \rangle$ such that

$$\mathcal{L}(M_7) = \mathcal{L}(a^*ba^*)$$

   The definition of M_7 can be written in Ans as

```
M7 = <Sigma,Q,q0,F,delta>
```

   followed by the definitions of all components of the tuple.

   Write the states as `q0`, `q1`, etc., and give the function δ as a list of equations of the form `delta(q0,a) = ...`

8. What is the type of the transition function δ of a non-deterministic finite automaton? (This type describes its input/output behavior.)

   (a) $\delta : Q \times \Sigma \to Q$
   (b) $\delta : Q \times \Sigma \to \mathcal{P}(Q)$
   (c) $\delta : Q \times (\Sigma \cup \{\lambda\}) \to Q$
   (d) $\delta : Q \times (\Sigma \cup \{\lambda\}) \to \mathcal{P}(Q)$