Formal Reasoning 2025
Test Block 1: Propositional and Predicate Logic
(25/09/22)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

Propositional logic

1. Consider the English sentence:

   You don't get wet unless it rains, in which case you only get wet if you're outside.

   We want to formalize the meaning of just the first part

   'You don't get wet unless it rains'

   as a formula of propositional logic. For the dictionary, we use:

R	it rains
W	you get wet

   A good formalization of this part of the sentence is:

   (a) $R \to W$
   (b) $\neg R \to \neg W$
   (c) $(R \to W) \land (\neg R \to \neg W)$
   (d) $(R \to W) \lor (\neg R \to \neg W)$

   Hint: In general, the sentence 'You don't get wet unless it rains' is ambiguous. However, reading the provided context can help you determine what the meaning to be formalized is in this case.

2. Consider the truth table of the formula of propositional logic:

   $$\neg a \to b \leftrightarrow c$$

   For each column in the truth table, give the number of zeroes and ones.

   As an example of how to give these numbers, for the formula $a \lor \neg b$ the answer would be:

   ```
   a: 2 zeroes and 2 ones
   b: 2 zeroes and 2 ones
   ~b: 2 zeroes and 2 ones
   a \/ ~b: 1 zero and 3 ones
   ```

   Hint: So don't waste time on submitting the full truth table!

3. Which formula of propositional logic is logically equivalent to $\neg(a \to b)$?

(a) $\neg a \to \neg b$

(b) $\neg b \to \neg a$

(c) $a \land \neg b$

(d) $\neg a \land b$

4. Is it the case that for all propositional formulas f and g, that if $\not\models f$ and $\not\models g$, then also $\not\models f \lor g$?

(a) Yes, that is the case. Take for example $f := a$ and $g := b$. Then we find that $\not\models a$ and $\not\models b$, and indeed also $\not\models a \lor b$.

(b) Yes, that is the case, as is clear from the following table:

$\models f$	$\models g$	$\models f \lor g$
0	**0**	**0**
0	1	1
1	0	1
1	1	1

(c) No, that is not the case. In fact, the conclusion $\not\models f \lor g$ does not hold for *any* f and g that satisfy $\not\models f$ and $\not\models g$.

(d) No, that is not the case. The conclusion $\not\models f \lor g$ indeed holds for *some* f and g that satisfy $\not\models f$ and $\not\models g$, but there are counterexamples.

Predicate logic

5. Formalize as a formula of predicate logic the meaning of the English sentence:

Happiness is being loved.

Use for the dictionary:

B	the domain of beings
$H(x)$	x is happy
$L(x, y)$	x loves y

6. Consider the formula of predicate logic:

$$\forall x \in B\,[L(x, k) \land H(k)]$$

and consider the following two statements:

- The form of this formula, according to the official grammar from the course notes is:

$$(\forall x \in B\,\underline{(}L(x, k) \land H(k)\underline{)})$$

- The underlined parentheses in the formula in the previous statement belong to the syntax of the universal quantifier.

What is the case?

(a) Both statements are correct.

(b) Only the first statement is correct.

(c) Only the second statement is correct.

(d) Neither statement is correct.

7. There are formulas of predicate logic that can only be true in infinite models. An example of such a formula is:

$$[\forall x, y, z \in D\,(R(x,y) \land R(y,z) \to R(x,z))] \land \\ [\forall x \in D\, \neg R(x,x)] \land \\ [\forall x \in D\, \exists y \in D\, R(y,x)]$$

   We consider here models where D is interpreted as the infinite set of natural numbers $\mathbb{N}$. Which interpretation in a model makes this formula true?

   (a) $(\mathbb{N}, <)$ where $R(x,y)$ is interpreted as $x < y$
   (b) $(\mathbb{N}, <)$ where $R(x,y)$ is interpreted as $y < x$
   (c) $(\mathbb{N}, \leq)$ where $R(x,y)$ is interpreted as $x \leq y$
   (d) $(\mathbb{N}, \leq)$ where $R(x,y)$ is interpreted as $y \leq x$

8. We want to formalize the following English sentence as a formula of predicate logic:

   Only Sharon is not happy.

   We use for the dictionary:

B	the domain of beings
s	Sharon
$H(x)$	x is happy

   What is a good formalization for this?

   (a) $\neg H(s)$
   (b) $\forall x \in B\, [\neg H(x) \to x = s]$
   (c) $\neg \exists x \in B\, [\neg H(x) \land x \neq s]$
   (d) none of the above

   Hint: Keep in mind that according to the sentence, Sharon is not happy.