Formal Reasoning 2025
Test Block 2: Discrete mathematics
(16/10/25)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. Good luck!

Discrete mathematics

1. Does each connected graph in which each vertex has even degree have an Eulerian circuit?

   (a) Yes, that is exactly the statement about Eulerian circuits in Euler's theorem.

   (b) Yes, but it also has an Eulerian circuit in the case when at most two vertices have an odd degree.

   (c) No, the Petersen graph is a counterexample.

   (d) No, the graph K_1 is a counterexample.

2. Is every tree bipartite?

   (a) No, because a graph can be bipartite and have a cycle (for example, the cube graph), which means there are bipartite graphs that are not trees.

   (b) No, because a graph can be bipartite and not be connected (for example, a graph with two vertices and no edges), which means there are bipartite graphs that are not trees.

   (c) Yes, because all connected graphs are bipartite, which means that all trees are bipartite.

   (d) Yes, because all graphs without a cycle are bipartite, which means that all trees are bipartite.

3. Give a graph that has an Eulerian circuit, but does not have a Hamiltonian circuit.

   Write the graph as $\langle V, E\rangle$ by giving V and E using set notation in the style:

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   V = { ... }
   E = { ... }
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4. We define the function a with two arguments using the recursion equations:

   $$a(m,0) = m$$
   $$a(m,k+1) = a(m,k) + 1$$

   In this, both m and k range over all natural numbers.

   What is the value of $a(3,4)$?

   (a) 3

(b) 4

(c) 7

(d) none of the above

5. Recall that we defined the function a with two arguments using the recursion equations:

$$a(m, 0) = m$$
$$a(m, k+1) = a(m, k) + 1$$

In this, again, both m and k range over all natural numbers.

Prove by induction that $a(0, n) = n$ for all natural numbers n.

Note: Although this exercise and the previous one are both about the same function, the result of this exercise is not usable for solving the previous exercise, because $a(3, 4)$ is not of the form $a(0, n)$.

6. In the induction step of an induction proof, there occurs both a statement $P(k)$ and a statement $P(k+1)$. What is the case?

   (a) The statements $P(k)$ and $P(k+1)$ both may be used in the part of the induction step that starts with 'This indeed holds, because ...'

   (b) The statements $P(k)$ and $P(k+1)$ both have to be shown to be true in the part of the induction step that starts with 'This indeed holds, because ...'

   (c) The statement $P(k)$ may be used, but the statement $P(k+1)$ has to be shown in the part of the induction step that starts with 'This indeed holds, because ...'

   (d) The statement $P(k)$ has to be shown, but the statement $P(k+1)$ may be used in the part of the induction step that starts with 'This indeed holds, because ...'

7. What is the number of edges in the graph K_n, for $n \geq 2$?

   (a) $\binom{n-1}{2}$

   (b) $\binom{n}{2}$

   (c) $\binom{n+1}{2}$

   (d) none of the above

8. If we know that the number of ways to choose three elements out of five objects is 35, then it must be the case that:

   (a) The order does matter and duplicates are allowed.

   (b) The order does matter, but duplicates are not allowed.

   (c) The order does not matter, but duplicates are allowed.

   (d) The order does not matter and duplicates are not allowed.