Formal Reasoning 2025
Test Block 3: Languages
(24/11/25)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. Good luck!

Languages

1. Which of the following four languages is different from the other three?

   (a) $(\{a, b\})^*$
   (b) $(\{a, b, a\})^*$
   (c) $(\{a\}^* \cup \{b\}^*)$
   (d) $(\{a\} \cup \{b\})^*$

2. Let

$$\begin{array}{rcl} \Sigma & := & \{a, b\}, \\ L_2 & := & \{w \in \Sigma^* \mid w \text{ starts with the symbol } a\}, \text{ and} \\ L'_2 & := & \{w \in \Sigma^* \mid w \text{ starts with the symbol } b\}. \end{array}$$

   In which of the alternatives are all statements correct?

   (a) $L_2 \cap L'_2 \neq \varnothing$, $L_2^* = \Sigma^*$, $L_2 \cup L'_2 \neq \Sigma^*$, and $L_2^* \cup L_2'^* = \Sigma^*$.
   (b) $L_2 \cap L'_2 = \varnothing$, $L_2^* \neq \Sigma^*$, $L_2 \cup L'_2 = \Sigma^*$, and $L_2^* \cup L_2'^* \neq \Sigma^*$.
   (c) $L_2 \cap L'_2 = \varnothing$, $L_2^* \neq \Sigma^*$, $L_2 \cup L'_2 \neq \Sigma^*$, and $L_2^* \cup L_2'^* = \Sigma^*$.
   (d) $L_2 \cap L'_2 \neq \varnothing$, $L_2^* = \Sigma^*$, $L_2 \cup L'_2 = \Sigma^*$, and $L_2^* \cup L_2'^* \neq \Sigma^*$.

3. Consider the language

$$L_3 := \big\{w \in \{a, b, c\}^* \mid w \text{ does not contain } ab\big\}$$

   So *aacbbacbca* and *bbccaaccbb* are elements of L_3, but *bbcaabcacb* is not. Here are two claims about this language:

   1. $L_3 = \mathcal{L}((aa^*c \cup b \cup c)^*a^*)$
   2. $L_3 = \mathcal{L}((b^*a^*c)^*b^*a^*)$

   Now what is the case?

   (a) Both claims are incorrect.
   (b) Claim 1 is correct, but claim 2 is incorrect.
   (c) Claim 1 is incorrect, but claim 2 is correct.
   (d) Both claims are correct.

   Hint: Try to recognize the structure of the expression(s) in the examples.

4. Consider the language

$$L_4 = \mathcal{L}\left(\big(a(a \cup b) \cup b\big)\big((a \cup b)(a \cup b)\big)^*\right)$$

Which words are elements of L_4?

   (a) $aabbaa$
   (b) $baaaaab$
   (c) $ababbaba$
   (d) All of the above.

5. Consider the grammar

$$G_5 = \langle \{a, b\}, \{S, B\}, \{S \to B, S \to aB, B \to bS, B \to \lambda\}\rangle$$

What is the minimum number of steps in a production for the word $abab$?

   (a) 4 or less
   (b) 5
   (c) 6
   (d) 7 or more

6. Consider the grammar G_6 given by the following rules:

$$\begin{array}{lcl} S & \to & A \mid aA \\ A & \to & bS \mid BS \mid \lambda \\ B & \to & bB \end{array}$$

We want to prove that $baaab \notin \mathcal{L}(G_6)$. Someone claims that this can be done with the invariant

$$P_6(w) := [\, w \text{ does not contain } aS \text{ and } w \text{ does not contain } aa \,]$$

Will this work? Explain your answer.

7. Consider the grammar G_7 given by the following rules:

$$\begin{array}{lcl} S & \to & aA \mid aB \\ A & \to & aS \\ B & \to & BB \mid b \mid \lambda \end{array}$$

Now what is the case?

   (a) Grammar G_7 is right linear and $\mathcal{L}(G_7)$ is a regular language.
   (b) Grammar G_7 is not right linear and $\mathcal{L}(G_7)$ is a regular language.
   (c) Grammar G_7 is right linear and $\mathcal{L}(G_7)$ is not a regular language.
   (d) Grammar G_7 is not right linear and $\mathcal{L}(G_7)$ is not a regular language.

8. Consider the grammar G_8 given by the following rules:

$$\begin{array}{lcl} S & \to & AB \mid BCS \\ A & \to & aaA \mid C \\ B & \to & bB \mid bbC \\ C & \to & Cc \mid \lambda \end{array}$$

First, give the nullable nonterminals in G_8 and then give a grammar G'_8 without any λ-rules such that $\mathcal{L}(G_8) = \mathcal{L}(G'_8)$.