Formal Reasoning 2025
Test Block 4: Automata and Modal Logic

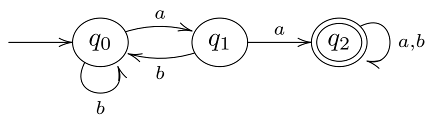
(18/12/25)

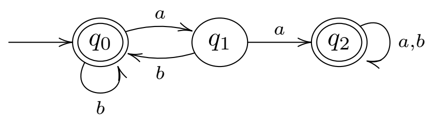
There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. Good luck!

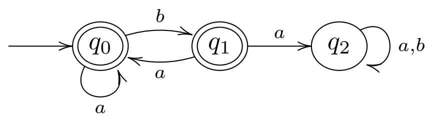
Automata

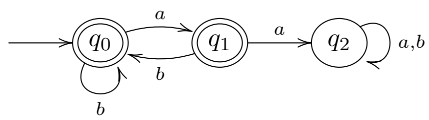
1. Which deterministic finite automaton accepts the language

$$L_1 := \{w \in \{a,b\}^* \mid w \text{ does not contain } aa\}$$

(a) 

(b) 

(c) 

(d) 

2. Consider the context-free grammar

$$G_2: \quad S \to aa \mid SbS$$

Give a deterministic finite automaton M_2 such that $\mathcal{L}(M_2) = \mathcal{L}(G_2)$. Provide the automaton by copying the list below and completing it by typing your answers in ASCII:

```
Set of states:
Initial state:
Set of final states:
delta(q0, a):
delta(q0, b):
```

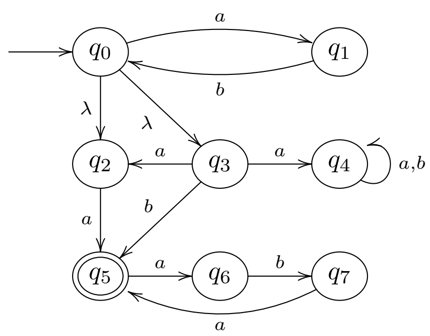
3. How many different deterministic finite automata M_3 over the alphabet $\Sigma := \{a, b\}$ exist, such that

- M_3 has at most two states,

- $\lambda \notin \mathcal{L}(M_3)$, and
- $ab \in \mathcal{L}(M_3)$.

(a) 0
(b) 4
(c) 8
(d) 12

4. Consider the following non-deterministic finite automaton M_4:



What is $\mathcal{L}(M_4)$?

(a) $\mathcal{L}((ab)^*(a \cup b)(aba)^*)$
(b) $\mathcal{L}((ab)^*(\lambda \cup a)(a \cup b)(aba)^*)$
(c) $\mathcal{L}((ab)^*(a \cup aa \cup b)(aba)^*)$
(d) None of the above.

Modal logic

5. Consider the sentence

Always when it rains, I get wet.

If we translate this sentence to a formula using the modality of 'time' and the dictionary

R	it rains
W	I get wet

which formula is the best option?

(a) $\Box(R \to W)$
(b) $\Box R \to W$
(c) $R \to \Box W$
(d) $\Box R \to \Box W$

6. In the logic T, the axiom schemes $\Box f \to f$ and $\Box f \to \Diamond f$ both hold for all formulas f. Is this logic appropriate for deontic logic?

(a) No, because it is possible to require something that is not forbidden.

(b) No, because it is possible not to do things that are obligatory.

(c) Yes, because it is not possible not to do things that are obligatory.

(d) Yes, because it is not possible to require something that is forbidden.

7. Provide a Kripke model $\mathcal{M}_7$ with at most three worlds $\{x_0, x_1, x_2\}$, which is serial, not reflexive, and such that the formula $\Box a \leftrightarrow \Diamond a$ holds, or more precisely, such that

$$\mathcal{M}_7 \vDash \Box a \leftrightarrow \Diamond a$$

Provide your answer by filling in the following table, which combines the definition of the model with a proof of correctness that your model satisfies the claim $\mathcal{M}_7 \vDash \Box a \leftrightarrow \Diamond a$.

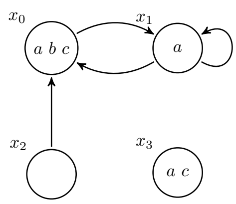
Do not forget to add a conclusion about what the table actually shows.

```
worlds ||     R      |      V      |  ||- a  |  ||- []a  |  ||- <>a  |  ||- []a <-> <>a
-----------------------------------------------------------------------------------
x0     ||            |             |         |           |           |
x1     ||            |             |         |           |           |
x2     ||            |             |         |           |           |
       ||            |             |         |           |           |
```

As Ans doesn't like 'verbatim' text, the table is given as a Python code block, but don't worry about the syntax highlighting, just type your answers in ASCII.

Please keep the bars to separate the columns, as that way we can still parse the table if its alignment is not perfect. Note that if your model has fewer than three worlds, you can just leave out the worlds that you don't need. And if it has more than three worlds, add rows, even though your model is not allowed to have more than three worlds.

Hint: Below you can find an example of a Kripke model and what such a table should look like.



worlds	R	V	$\Vdash a$	$\Vdash b$	$\Vdash c$	$\Vdash \Box a$	$\Vdash b \vee c$	$\Vdash \Diamond(b \vee c)$	$\Vdash \Box a \leftrightarrow \Diamond(b \vee c)$
x_0	$\{x_1\}$	$\{a, b, c\}$	1	1	1	1	1	0	0
x_1	$\{x_0, x_1\}$	$\{a\}$	1	0	0	1	0	1	1
x_2	$\{x_0\}$	$\{\}$	0	0	0	1	0	1	1
x_3	$\{\}$	$\{a, c\}$	1	0	1	1	1	0	0

8. Consider the sentence

Some day after tomorrow it will always be true that it never rains for three consecutive days.

Which LTL formula represents this sentence best, assuming that the time between x_i and x_{i+1} is a day? Use as a dictionary

R it rains

(a) $\mathcal{F}\mathcal{X}\mathcal{G}\,((R \wedge \mathcal{X}R) \rightarrow \neg\mathcal{X}\mathcal{X}R)$

(b) $\mathcal{X}\mathcal{F}\mathcal{X}\mathcal{G}\,((R \wedge \mathcal{X}R) \rightarrow \mathcal{X}\mathcal{X}\neg R)$

(c) $\mathcal{G}\mathcal{X}\mathcal{F}\,((R \wedge \mathcal{X}R) \rightarrow \mathcal{X}\mathcal{X}\neg R)$

(d) $\mathcal{G}\mathcal{X}\mathcal{X}\mathcal{F}\,((\neg R \wedge \neg\mathcal{X}R) \vee \neg\mathcal{X}\mathcal{X}R)$