Formal Reasoning 2025
Test Blocks 1, 2, 3 and 4: Additional Test
(07/01/26)

There are six multiple choice questions and two open questions. Each multiple choice question is worth 10 points, and the open questions are worth 15 points each. Good luck!

Propositional logic

1. The logical operator $\vee$ is a so-called *inclusive or*, as it is allowed that both of its operands are true. However, it is also possible to disallow that both operands are true, which is called an *exclusive or*, represented by the symbol $\oplus$, and its truth table is defined as follows:

a	b	$a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0

The formula $a \oplus b$ should be read as 'a, or b, but not both'.

Which of the following formulas is *not* logically equivalent to $a \oplus b$?

(a) $a \wedge (\neg b) \vee (\neg a) \wedge b$
(b) $a \vee b \wedge \neg(a \wedge b)$
(c) $\neg a \leftrightarrow b$
(d) $\neg(a \leftrightarrow b)$

Hint: Did you parse the formulas correctly with respect to binding strength and associativity?

Predicate logic

2. Consider the structure $M := (\mathbb{N}, -)$ and the interpretation I defined by:

N	$\mathbb{N}$
$S(x, y, z)$	$x - y = z$

Which of the following statements does not hold?

(a) $(M, I) \vDash \forall x \in N\, \exists y \in N\, S(x, y, y)$
(b) $(M, I) \vDash \forall x \in N\, \exists y \in N\, S(x, x, y)$
(c) $(M, I) \vDash \forall x \in N\, \exists y \in N\, S(x, y, x)$
(d) $(M, I) \vDash \forall x \in N\, \exists y \in N\, S(y, x, x)$

3. Translate into a formula of predicate logic:

The course Introduction to Formal Reasoning has exactly two lecturers and both are not only teaching this course.

Use the dictionary:

L	the domain of lecturers
C	the domain of courses
i	the course Introduction to Formal Reasoning
$T(l, c)$	l teaches course c

Discrete mathematics

4. Recall Euler's theorem:

   In a connected graph with at least two vertices:

   (a) An Eulerian circuit exists if and only if every vertex has an even degree.

   (b) An Eulerian path exists if and only if there are at most two vertices of odd degree.

   So the theorem holds for connected graphs with at least two vertices. In general, it no longer holds if one of these requirements is omitted.

   Now give a graph which has an even degree for all vertices, such that either

   - it is connected and has at most one vertex,
   - or it is not connected and it has at least two vertices,

   and explain that your graph does not have an Eulerian *path*.

   Write the graph as $\langle V, E\rangle$ by giving V and E using set notation in the style:

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   V = {... }
   E = {... }
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   Hint: You don't have to prove that your graph indeed meets the requirements for one of the two types; only explain why it doesn't have an Eulerian path.

5. We want to count the number of ways that we can put n distinguishable objects into k indistinguishable but possibly empty groups, where both $n \geq 1$ and $k \geq 1$. In how many ways is this possible?

   (a) $\left[{n \atop k}\right]$

   (b) $\left[{n \atop 1}\right] + \left[{n \atop 2}\right] + \cdots + \left[{n \atop k}\right]$

   (c) $\left\{{n \atop k}\right\}$

   (d) $\left\{{n \atop 1}\right\} + \left\{{n \atop 2}\right\} + \cdots + \left\{{n \atop k}\right\}$

Languages

6. Consider the context-free grammar G_6:

   $$S \to aaSb \mid \lambda$$

   Someone claims that

   $$P_6(w) := [\text{the number of } a\text{'s in } w \text{ is two times the number of } b\text{'s}]$$

   is an invariant for this language. Is this correct, and why?

(a) Yes, because the predicate holds for S, and the first rule of this grammar increases the number of a's by two and the number of b's by one, and the second rule doesn't change the number of a's and b', so both rules keep the ratio between a's and b's the same.

(b) Yes, because each word of the language is of the form $a^{2n}b^n$, which satisfies this property.

(c) No. The word *abaaba* satisfies this property, but is not in the language produced by this grammar.

(d) No. As $P_6(S)$ does not hold.

Automata

7. Let be given a deterministic finite automaton $M := \langle \Sigma, Q, q_0, F, \delta \rangle$, with $F \neq Q$. Does it follow that $\mathcal{L}(M) \neq \Sigma$? You may assume that $q_1 \in Q \setminus F$.

(a) Yes, because in a DFA any state is reachable from q_0 in one or more steps, so all words w that end up in q_1 are in Σ^* but not in $\mathcal{L}(M)$.

(b) Yes, because in a DFA there must be at least one word w that ends up in this q_1. So for this word it holds that $w \notin \mathcal{L}(M)$, but $w \in \Sigma^*$.

(c) No, take the automaton $M := \langle \{a, b\}, \{q_0, q_1\}, q_0, \{q_0\}, \delta \rangle$ where $\delta(q, x) = q_0$ for all q in Q and all $x \in \Sigma$. Then all words in Σ^* end up in q_0 and are accepted. So $\mathcal{L}(M) = \Sigma^*$.

(d) No, because in an NFA, if there is a word w that ends up in q_1, a λ-transition can be used to go to a final state in a single step from q_1. So $\mathcal{L}(M) = \Sigma^*$.

Modal logic

8. In an LTL Kripke model $\mathcal{M} := \langle W, R, V \rangle$ we can both write

$$\mathcal{M} \vDash f \qquad \text{and} \qquad \mathcal{M}, x_i \Vdash g$$

for formulas f and g, and for worlds $x_i \in W$.

Is there a relation between these two notations in LTL?

(a) Yes, because $\mathcal{M} \vDash f$ if and only if for all $x_i \in W$ it holds that $\mathcal{M}, x_i \Vdash \mathcal{F}f$.

(b) Yes, because $\mathcal{M} \vDash f$ if and only if $\mathcal{M}, x_0 \Vdash \mathcal{G}f$.

(c) No, because $\mathcal{M} \vDash f$ states something about truth in all models, whereas $\mathcal{M}, x_i \Vdash f$ states something about truth in all worlds.

(d) No, because each single model $\mathcal{M}$ has an infinite number of worlds, so the 'model claim' $\mathcal{M} \vDash f$ can never be given by a single 'world claim' $\mathcal{M}, x_i \Vdash g$.