B Method

Proof assistants

May 16, 2017

Lucas Franceschino

What is B method?

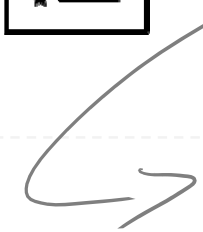
B-method goal

Specifications

Actual program

B-method goal

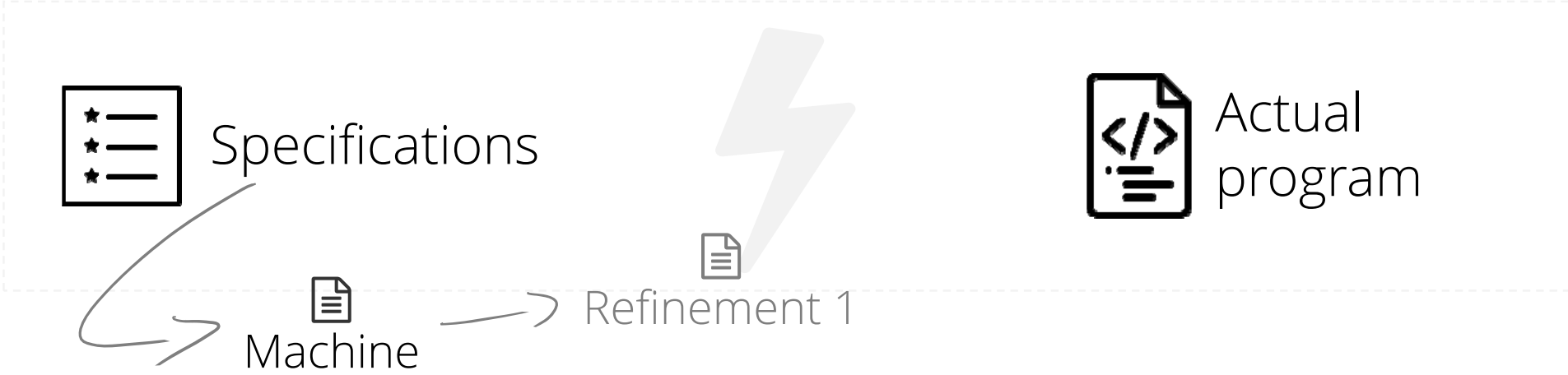
Specifications



Machine

Actual program

B-method goal



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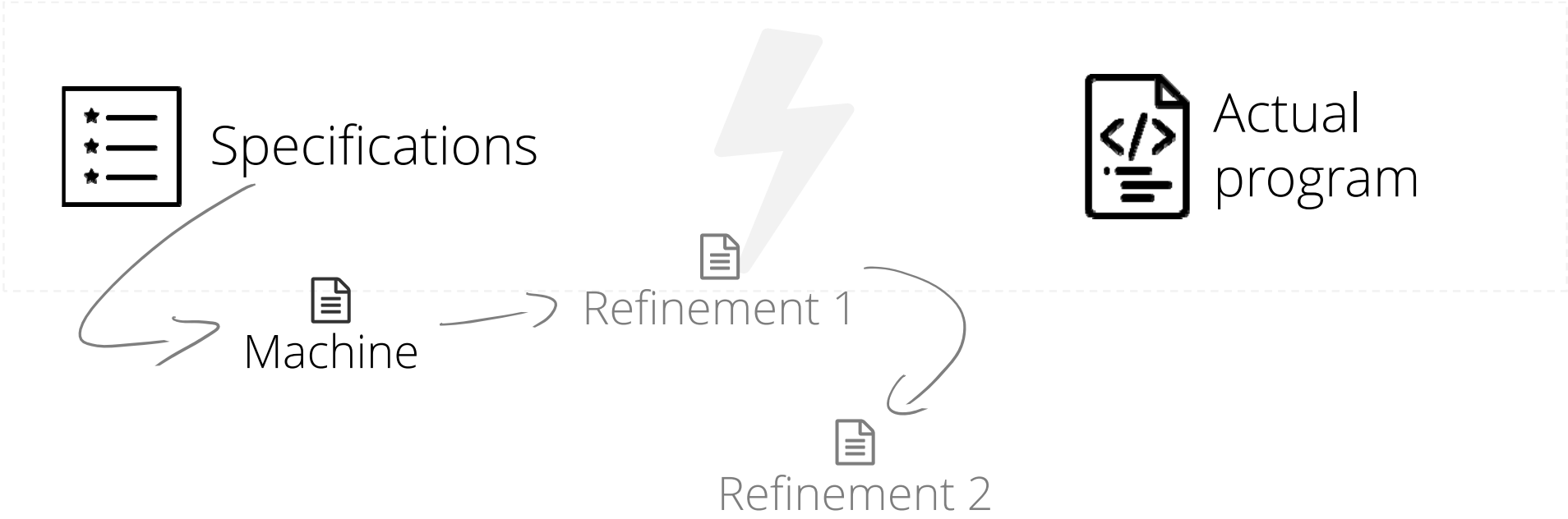

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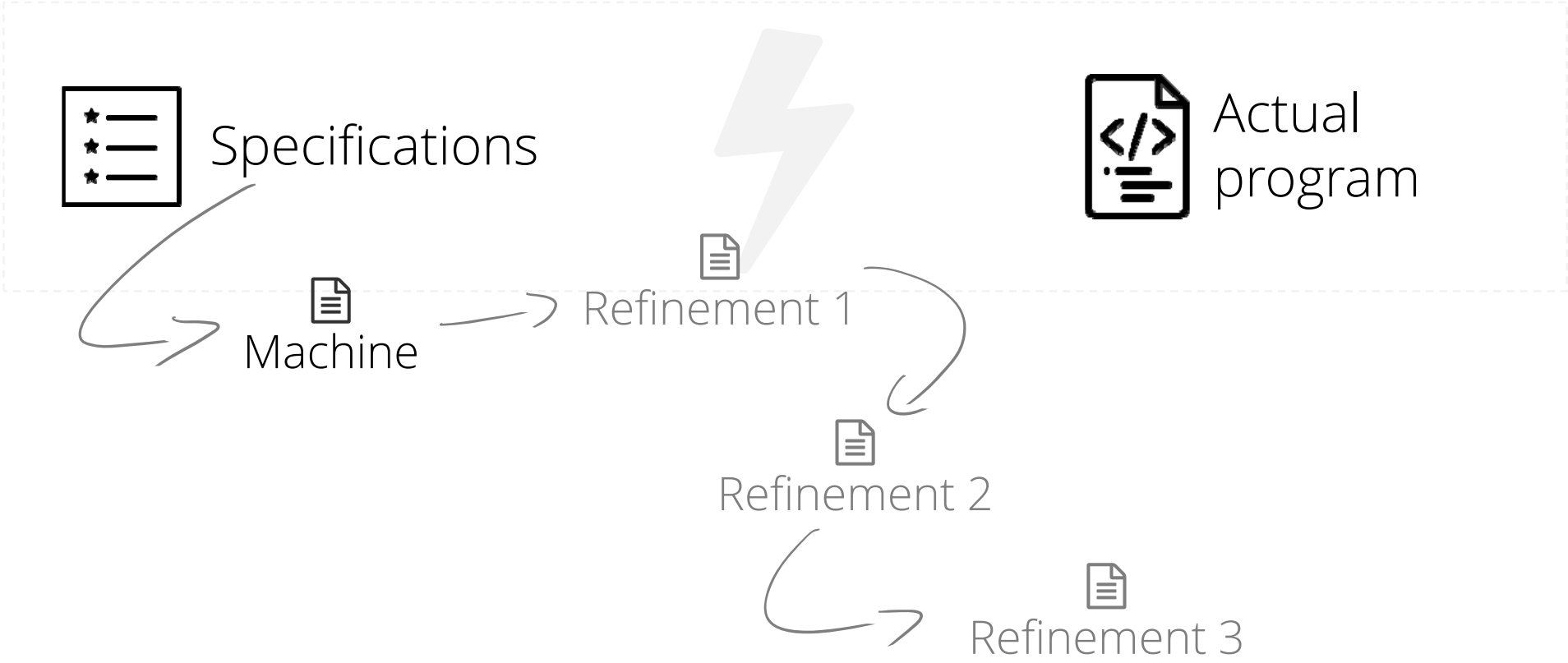

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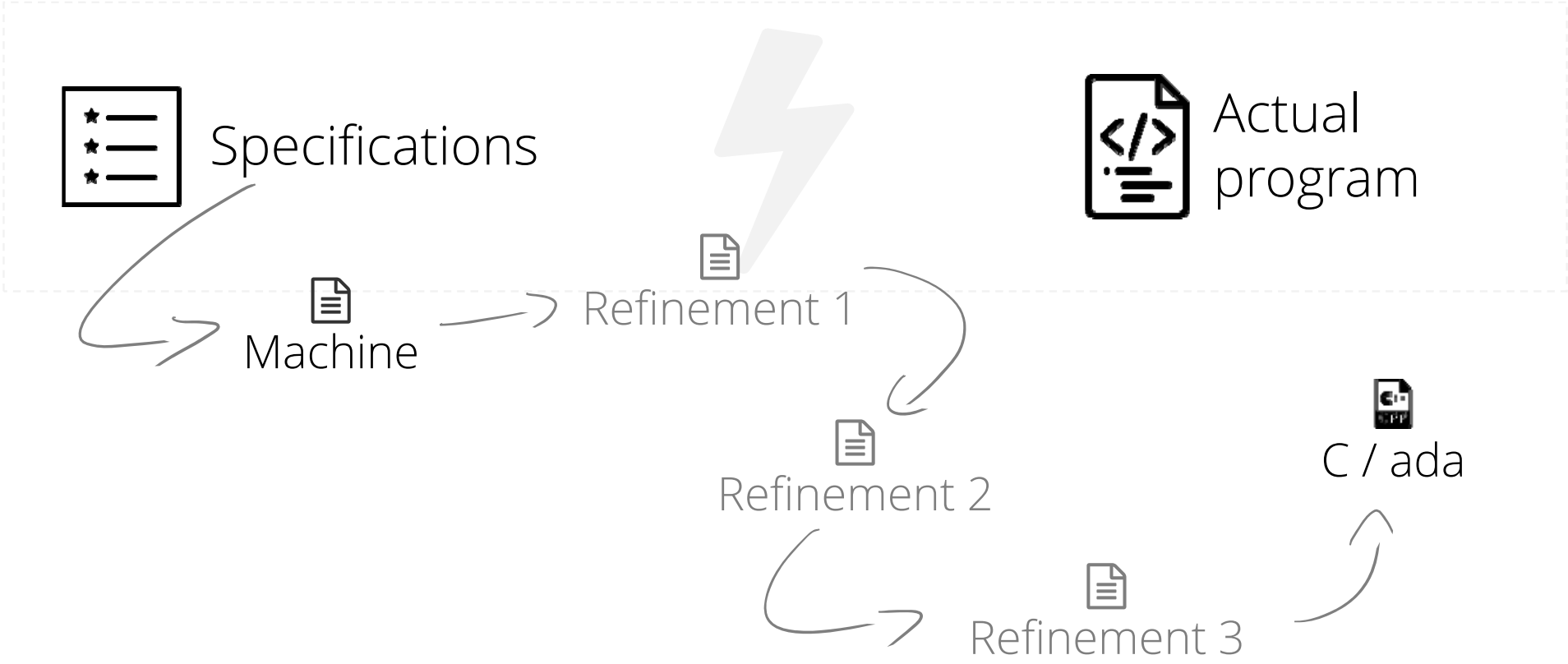

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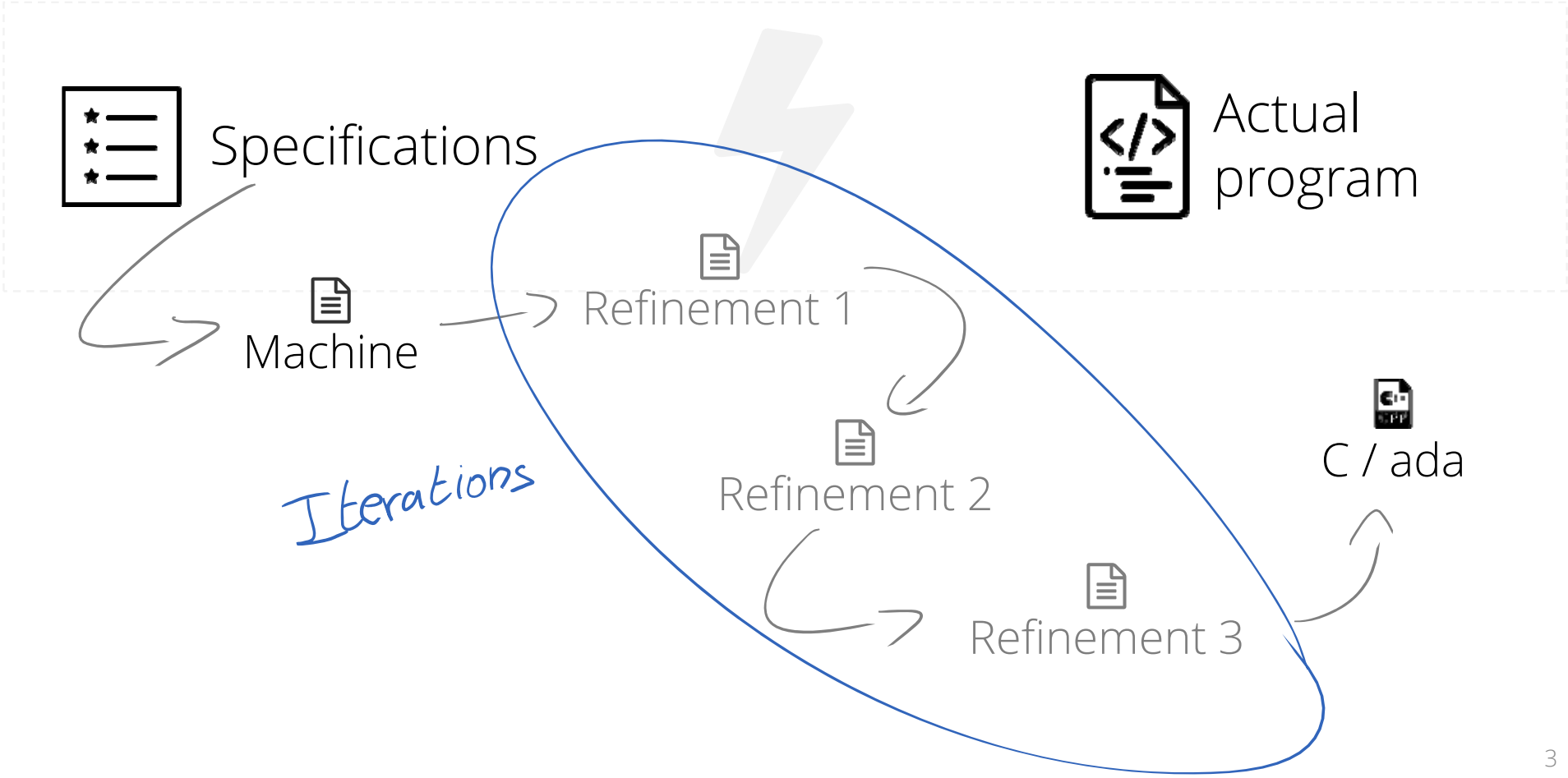

B-method goal

Specifications

Actual program

No gap between **specification** and **actual program**

The initiator of B method

Jean-Raymond Abrial

1970 Specification of data structures and programs

Initiated the Z-Notation (in Oxford)

→ *B4free, Bart, ABTools...*

G. Laffitte,
F. Mejia,
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Rodin platform

↳ Was in the development team of Ada

G. Laffitte,
F. Mejia,
I. Mc Neal

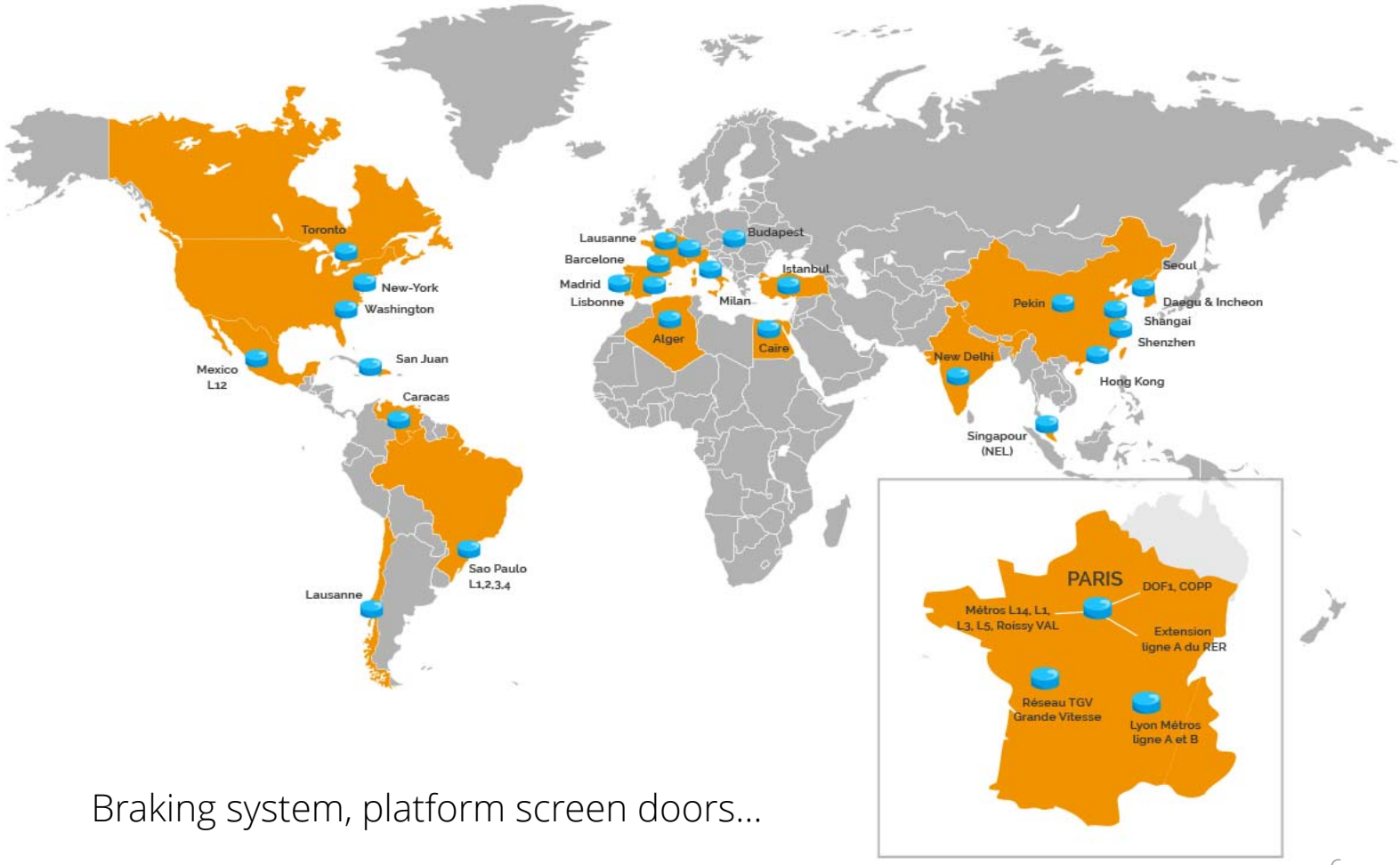
Use cases

Train related B projects around the world

CLEARSY
SYSTEMS ENGINEERING

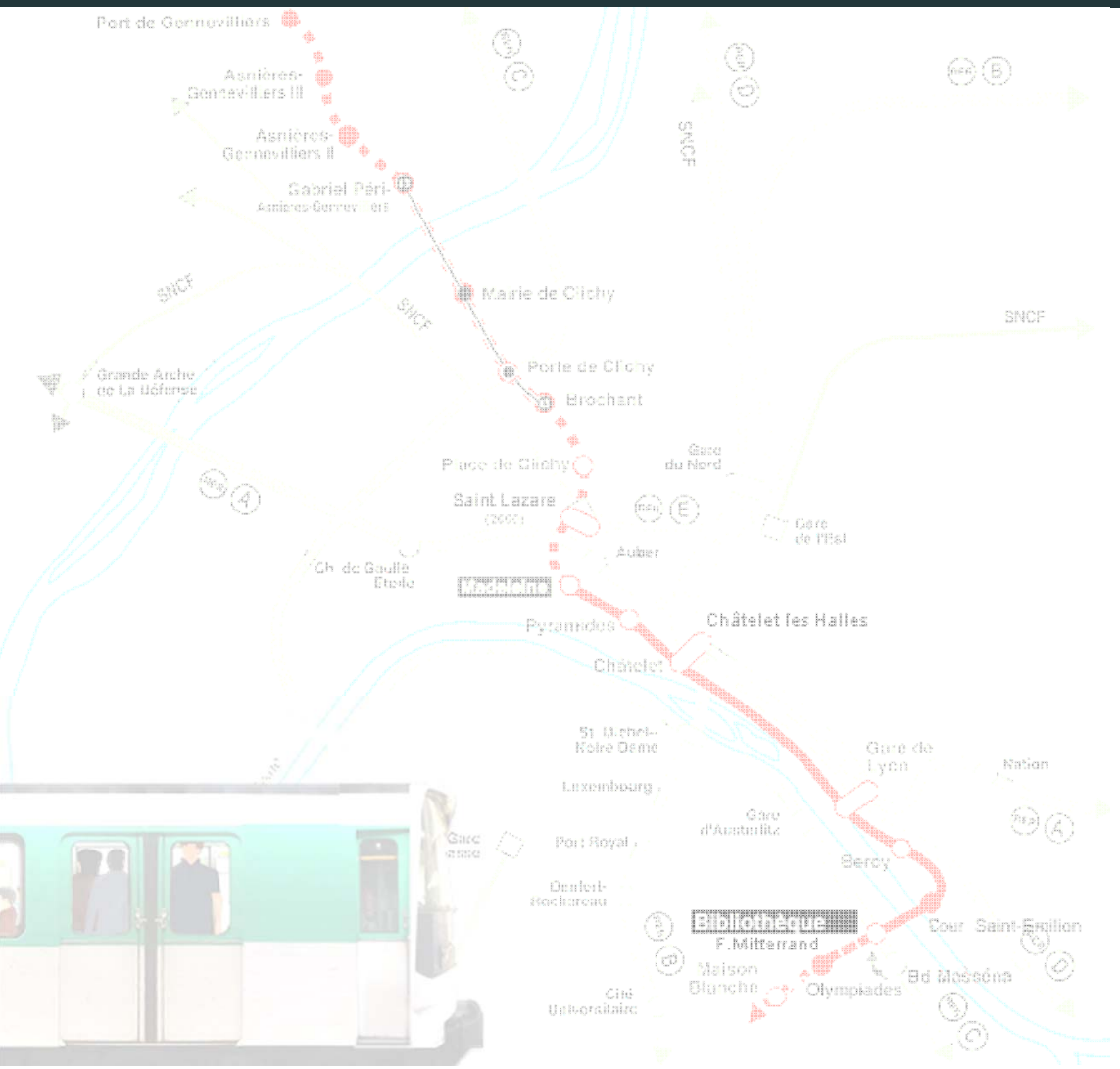
SIEMENS

ALSTOM



Braking system, platform screen doors...

Use case: Meteor in Paris (line 14)



110 000 lines of B </>

29 000 lemmas 👍

87 000 lines of Ada </>

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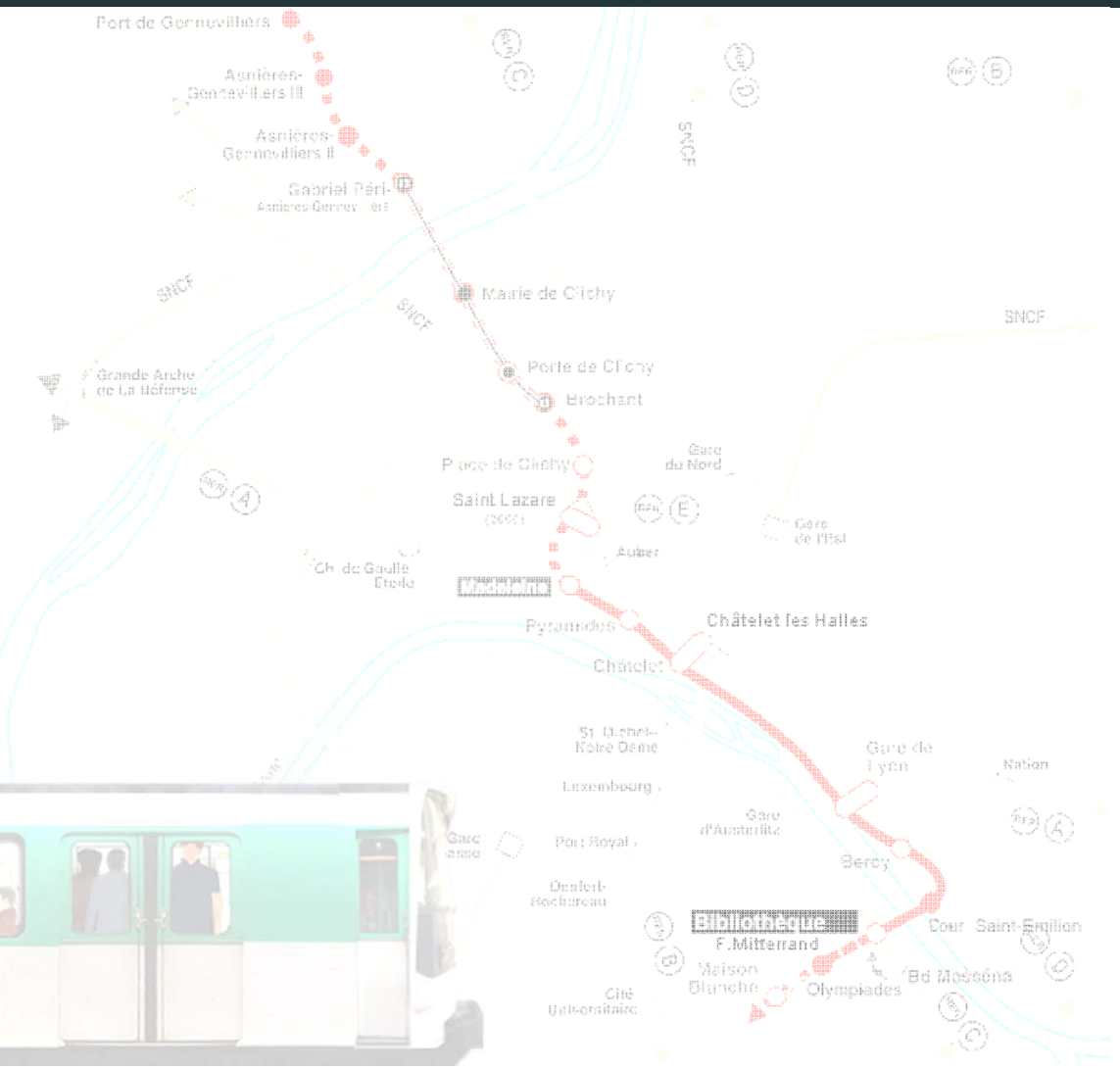

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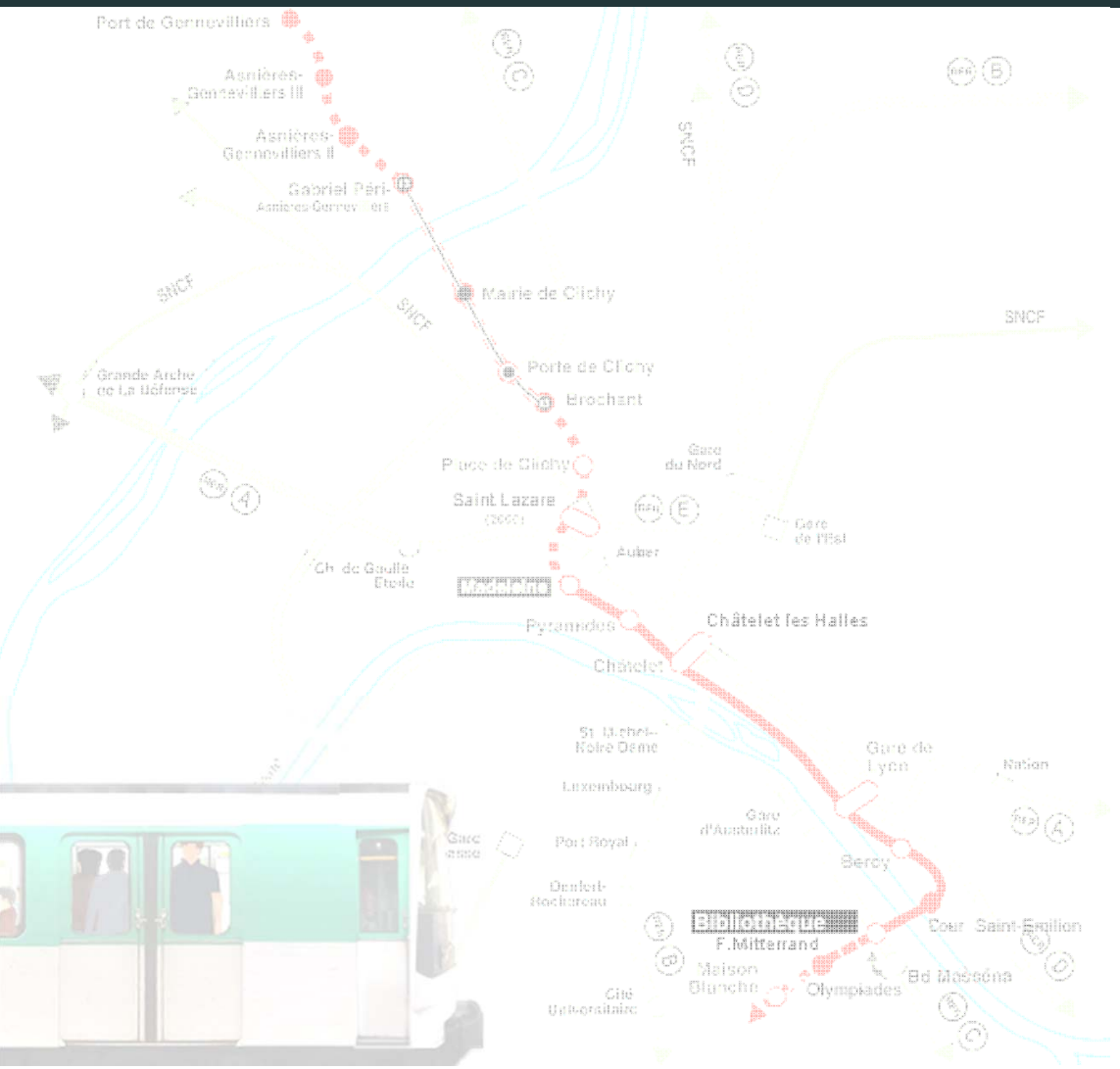
Driverless trains

Extension in 2003

80 million passengers in 2009

Use case: Meteor in Paris (line 14)



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9.2 km

October 1998

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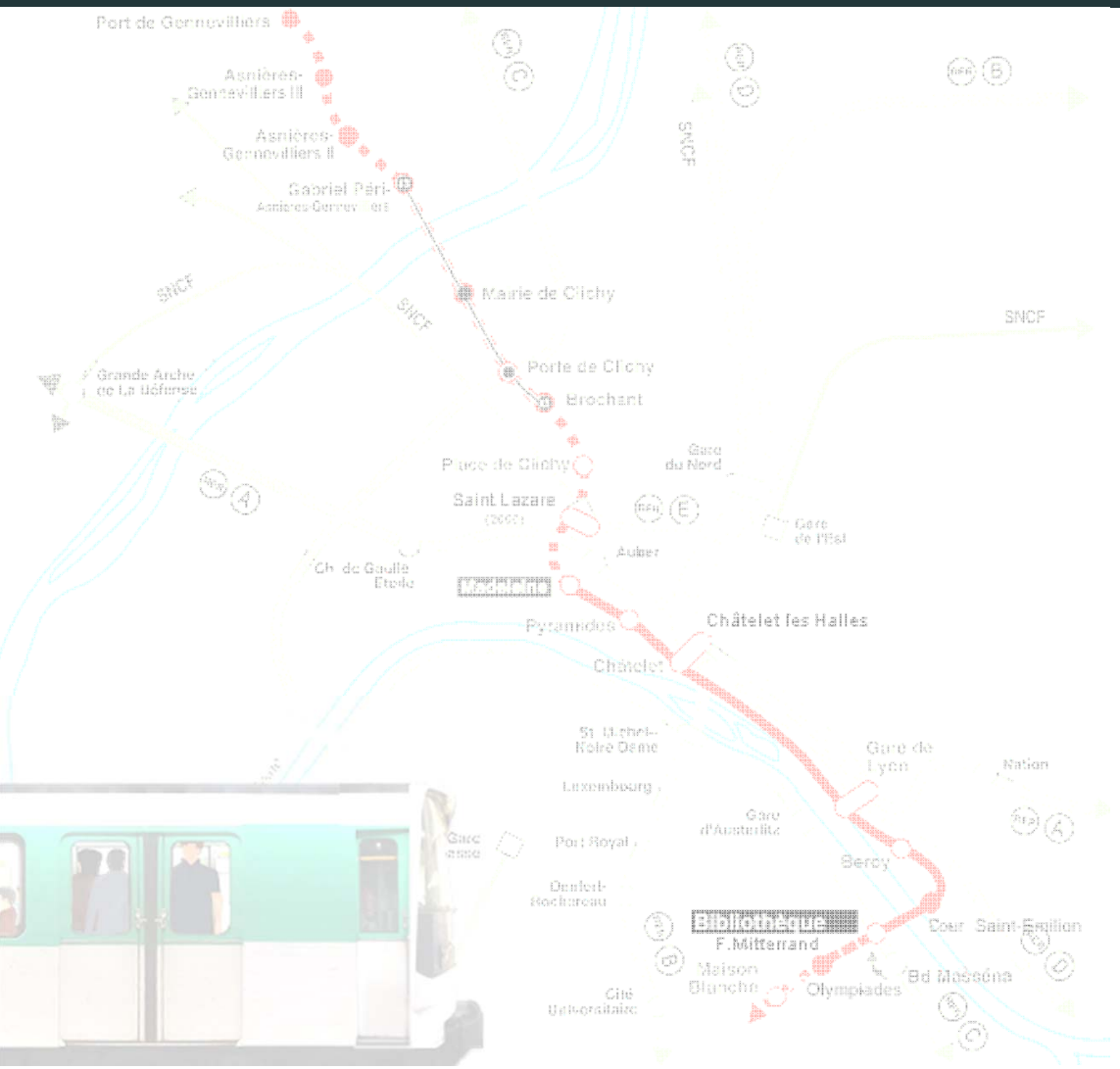

110 000 lines of B </>

29 000 lemmas

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9.2 km

October 1998

No bugs discovered yet!

Still in version 1.0

Driverless trains

Extension in 2003

80 million passengers in 2009

Other use cases

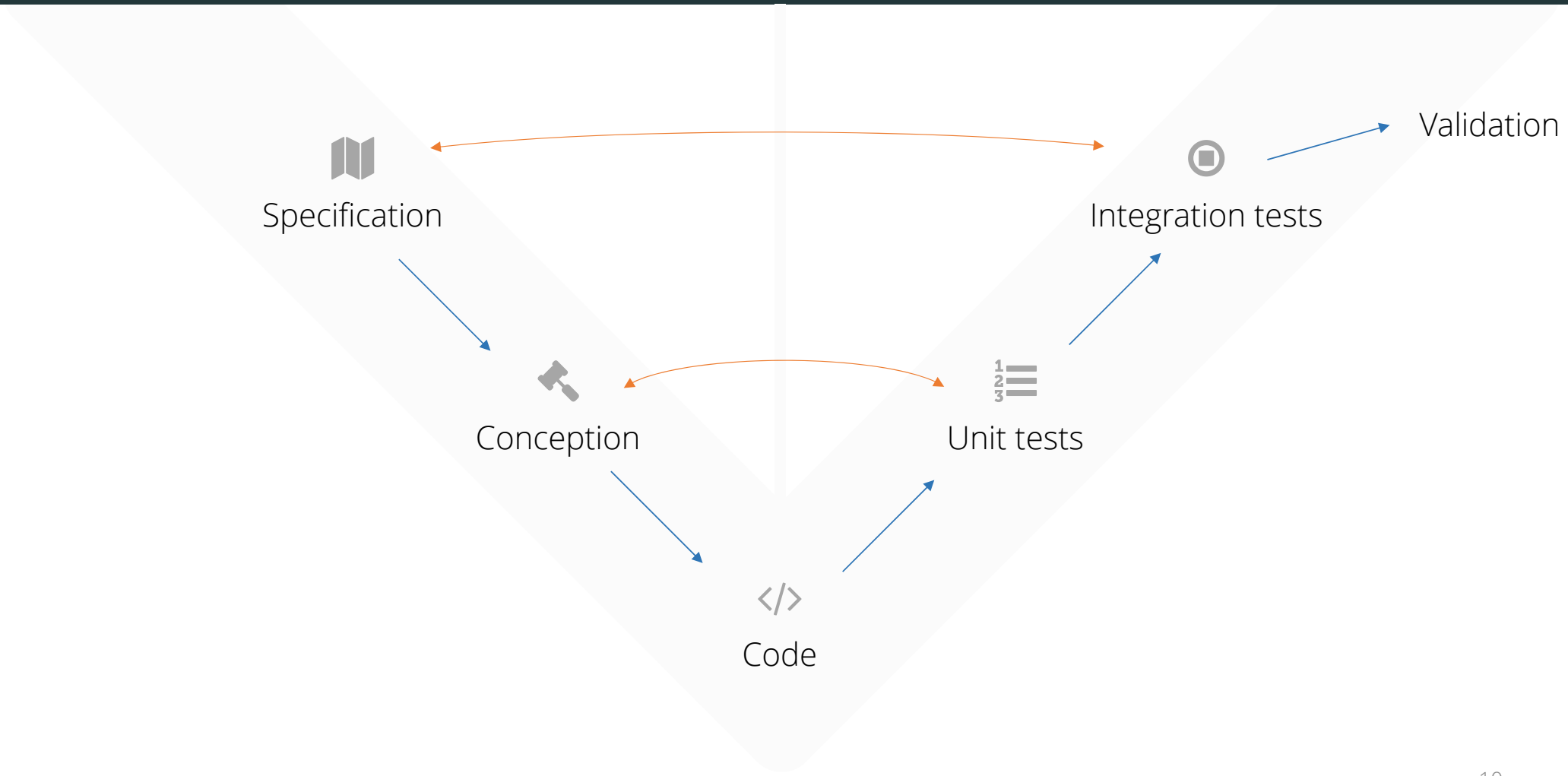
- Peugeot cars: formalization of sub systems (lights system, airbags, motor) to help building diagnostic tools
- Modeling of tasks scheduling from the software controlling the stage separations of Ariane rocket
- Protocol study
- JavaCard runtime formalization

  Java runtime for *smartcard*

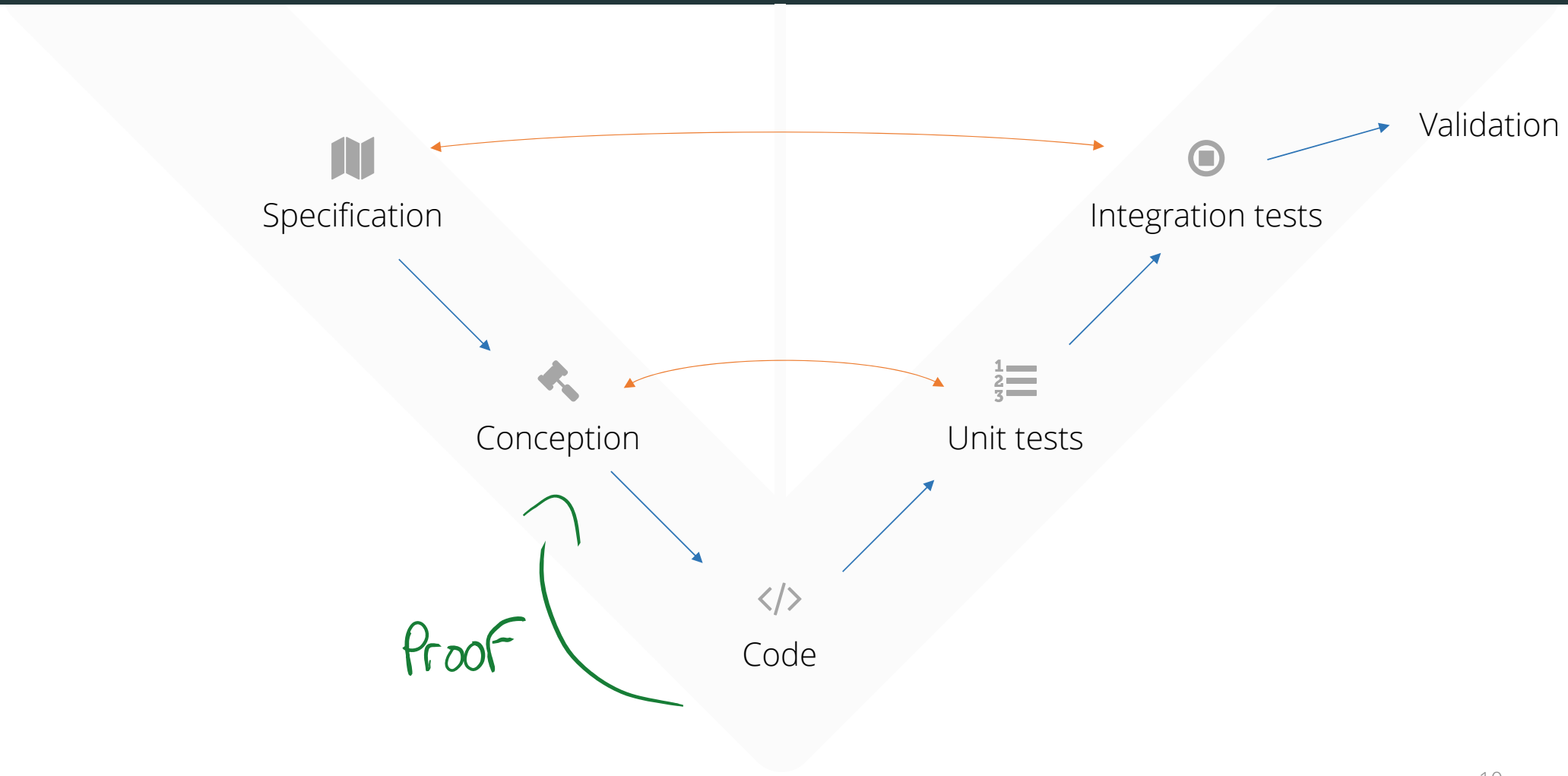
  ↳ Provide *safe:* authentication, data storage, application processing

Developing in B

B method development



B method development



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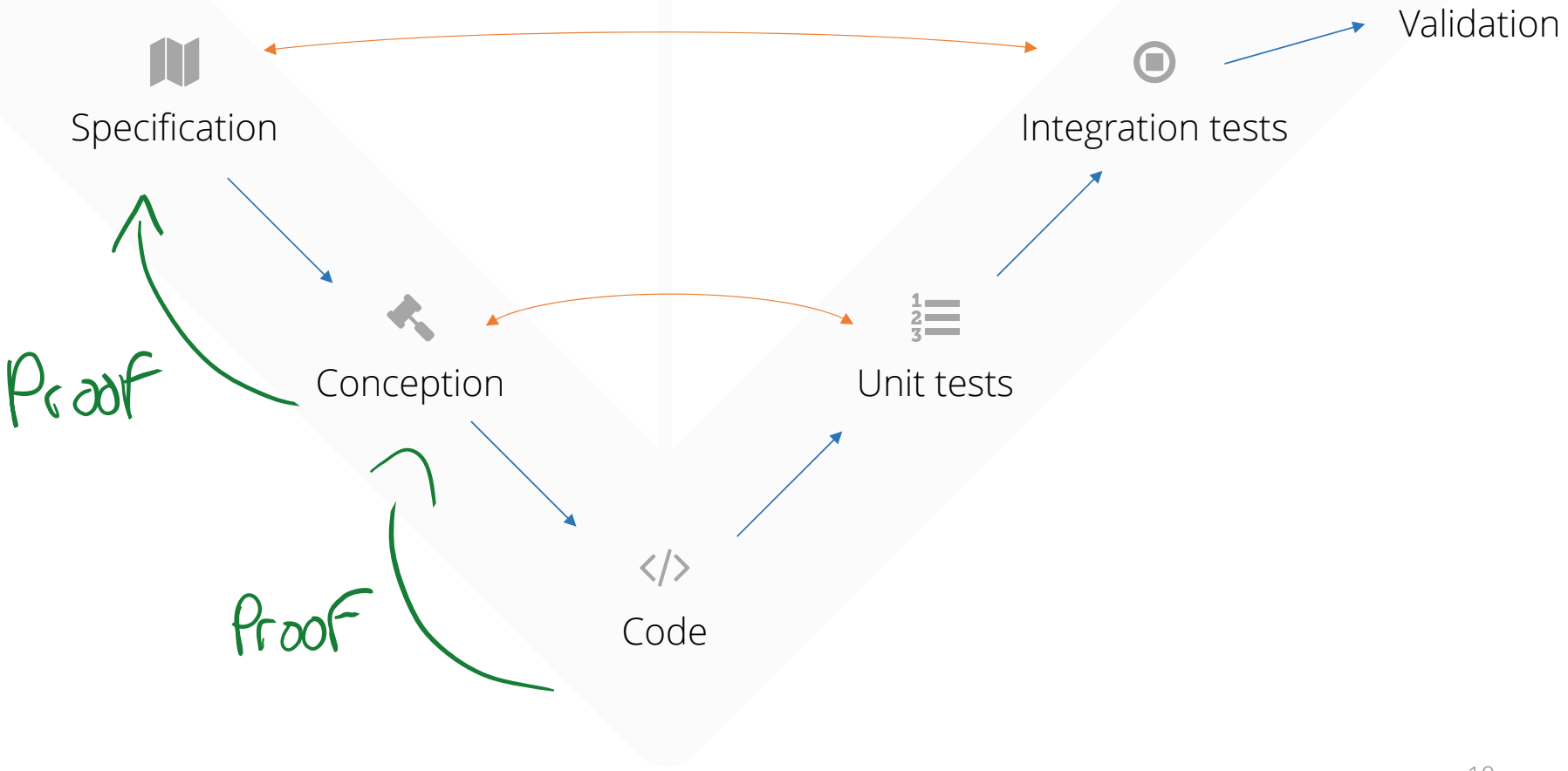

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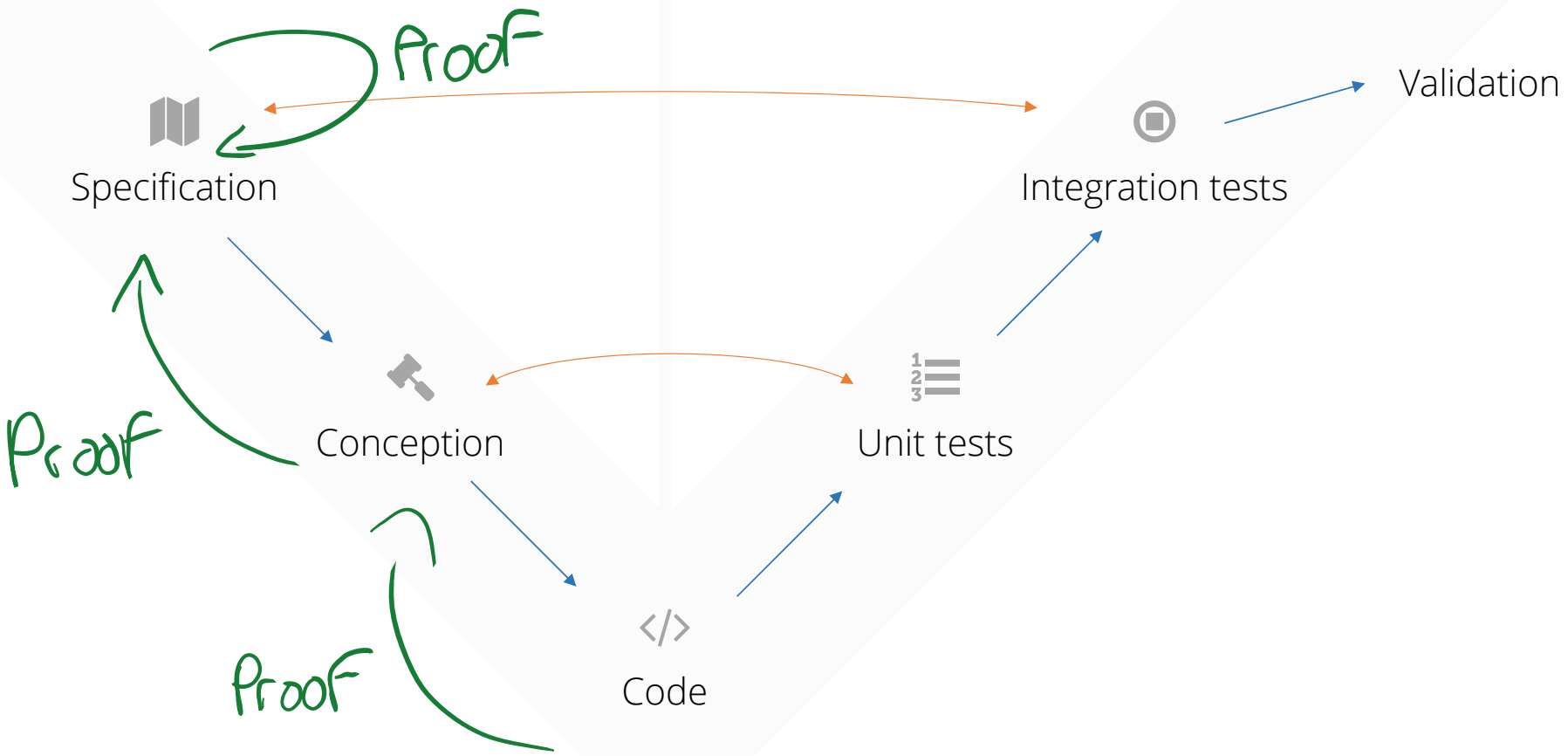

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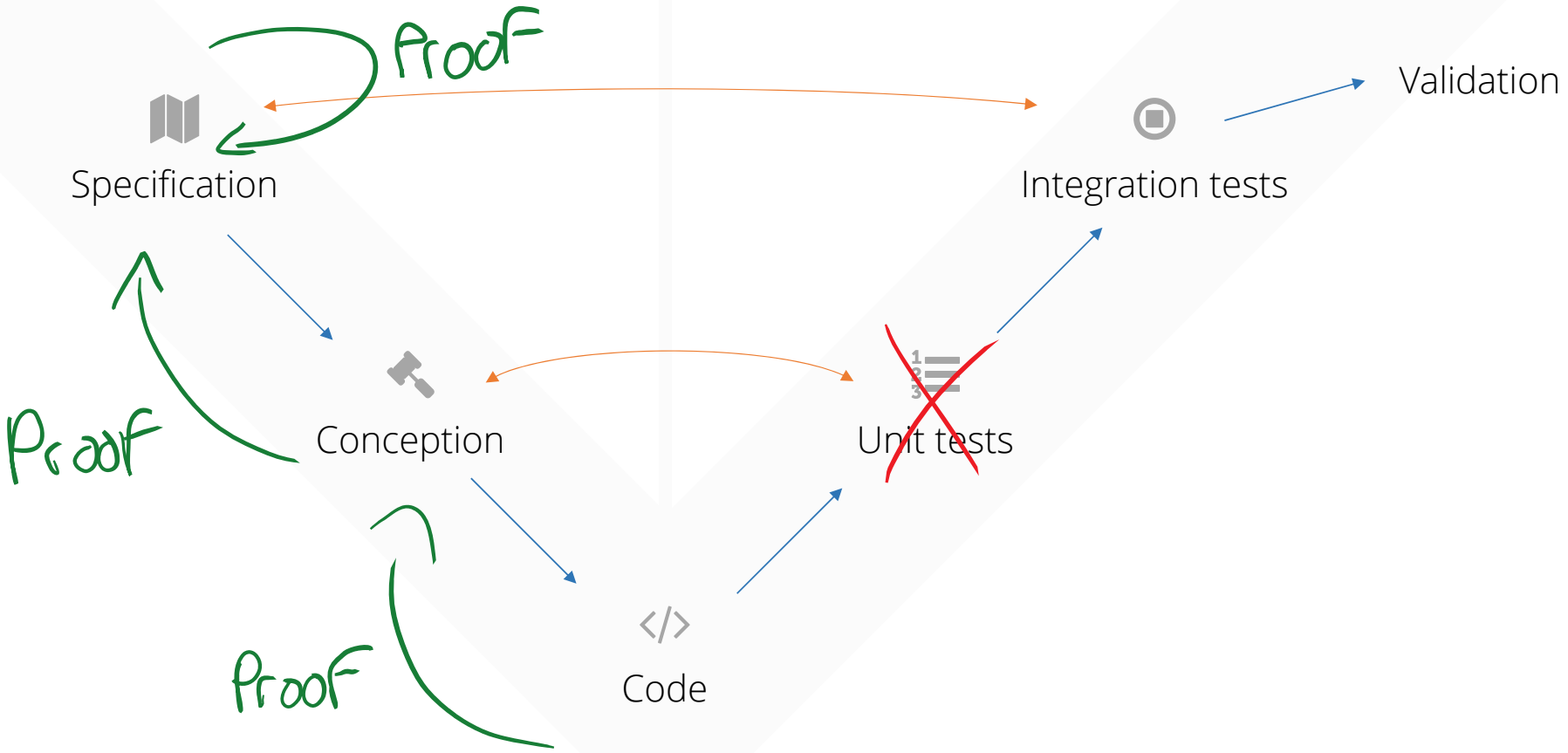

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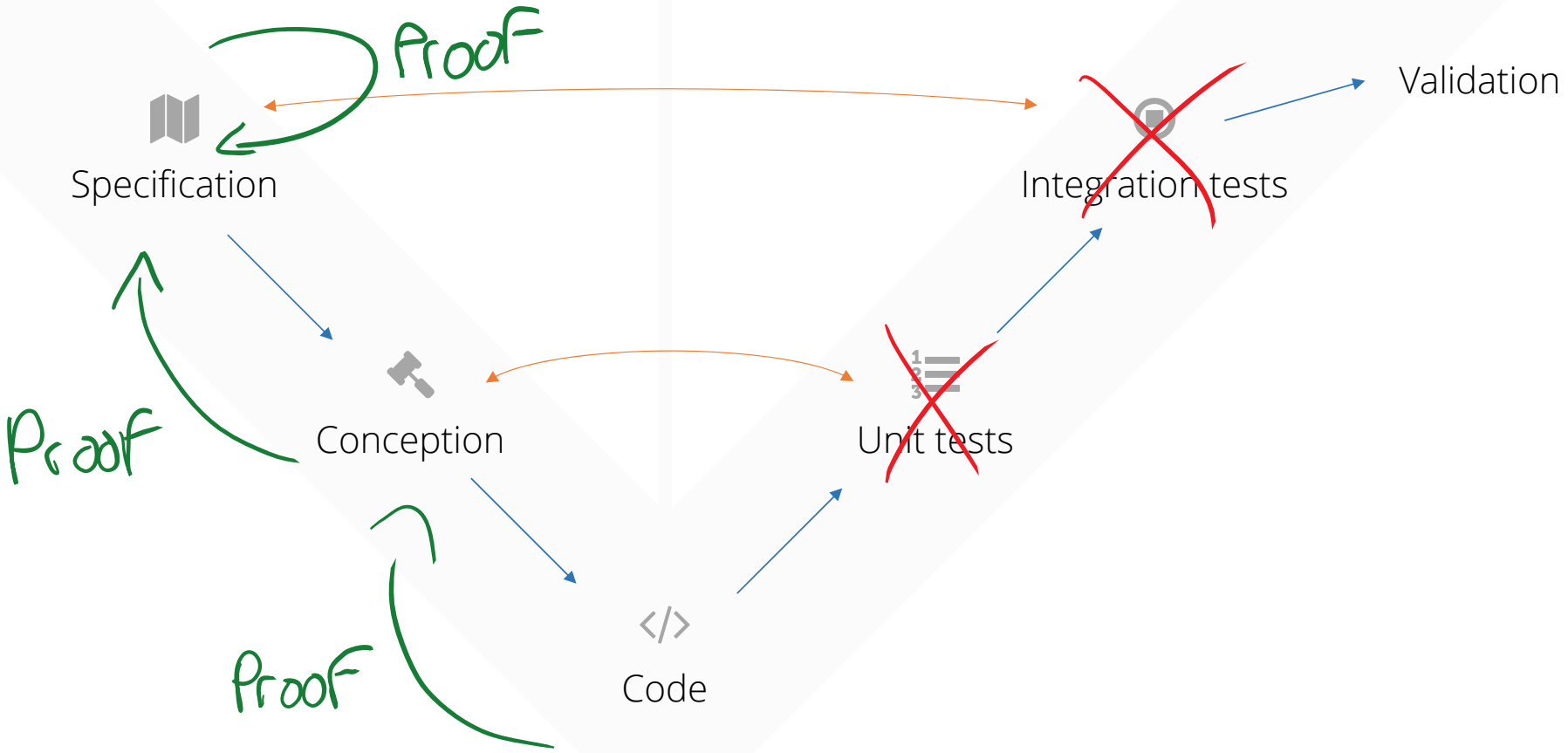

How does method B works

- Module: *modelling of a sub system*
  - Component

How does method B works

- Module: *modelling of a sub system*

- Component

Static part
- *Definition of:* Variables, constants, sets
- List of invariants

Dynamic part
- Initialize variables
- Define operations on variables

Proof
- Static part coherence
- Initializing preserve invariants
- Operations preserve invariants

How does method B works

Module: *modelling of a sub system*

Abstract machine

Component

Static part
- *Definition of:* Variables, constants, sets
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How does method B works

Module: *modelling of a sub system*

Abstract machine
Refinements

Component

Static part
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How does method B works

Module: *modelling of a sub system*

- Abstract machine
- Refinements
- Implementation

→ Component →

Static part

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How does method B works

Module: *modelling of a sub system*

- Abstract machine
- Refinements
- Implementation

Component

We refine a previous component: we make it more precise and specific

Static part

Definition of: Variables, constants, sets
List of invariants

Dynamic part

Initialize variables
Define operations on variables

Proof

Static part coherence
Initializing preserve invariants
Operations preserve invariants

Machine, refinement, implementation

- Abstract machine
- Refinements
- Implementation

Substitutions

Machine, refinement, implementation

- Abstract machine
- Refinements
- Implementation

Substitutions

Machine, refinement, implementation

Abstract machine

- ✓ Precondition
- ✓ Choice
- ✓ Let bindings
- ✗ Sequencing
- ✓ Become such that
- ✓ Simultaneous operations
- ✗ While loop

Refinements

Implementation

Substitutions

Machine, refinement, implementation

Abstract machine

Predicated-based IF

- Precondition
- Choice
- Let bindings
- Sequencing
- Become such that
- Simultaneous operations
- While loop

Refinements

Implementation

Substitutions

Machine, refinement, implementation

Abstract machine

Predicated-based IF

ANY x WHERE x ∈ S

- Precondition
- Choice
- Let bindings
- Sequencing
- Become such that
- Simultaneous operations
- While loop

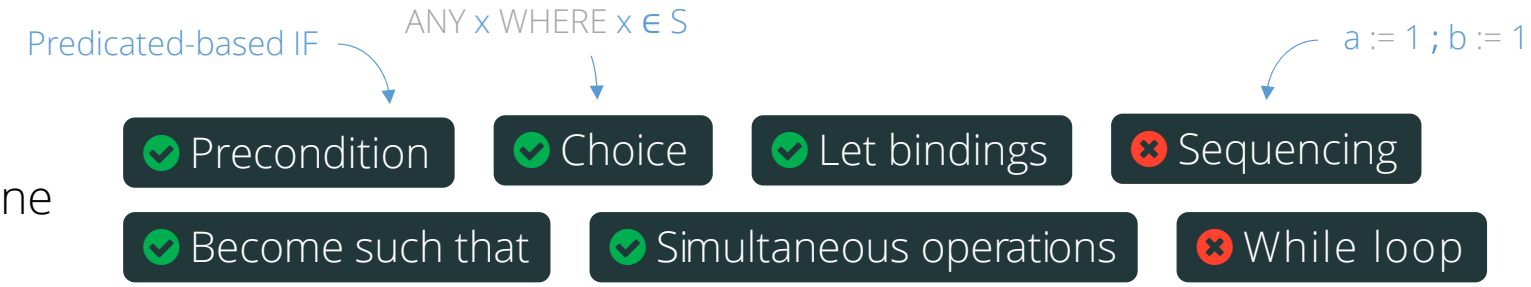
Refinements

Implementation

Substitutions

Machine, refinement, implementation

Abstract machine



Refinements

Implementation

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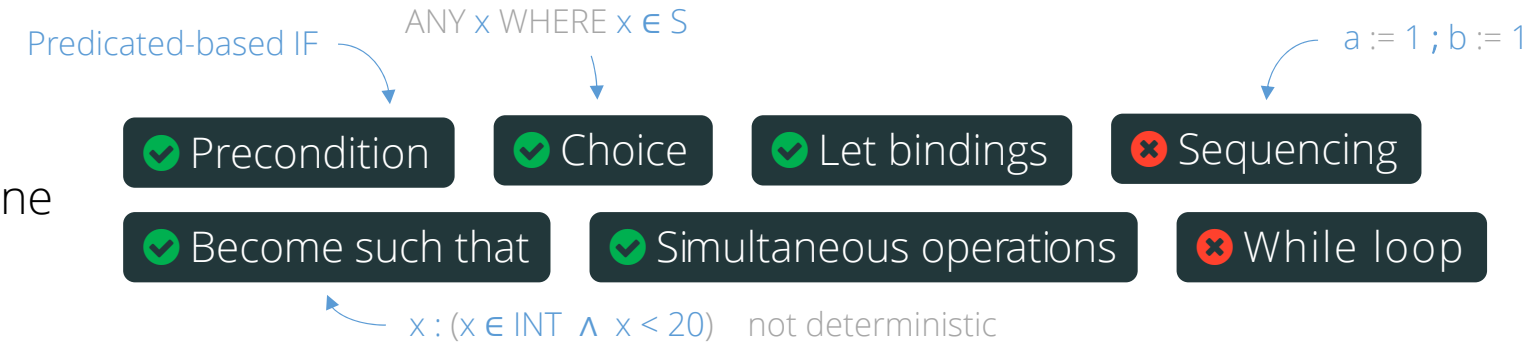
Machine, refinement, implementation

Abstract machine



Refinements

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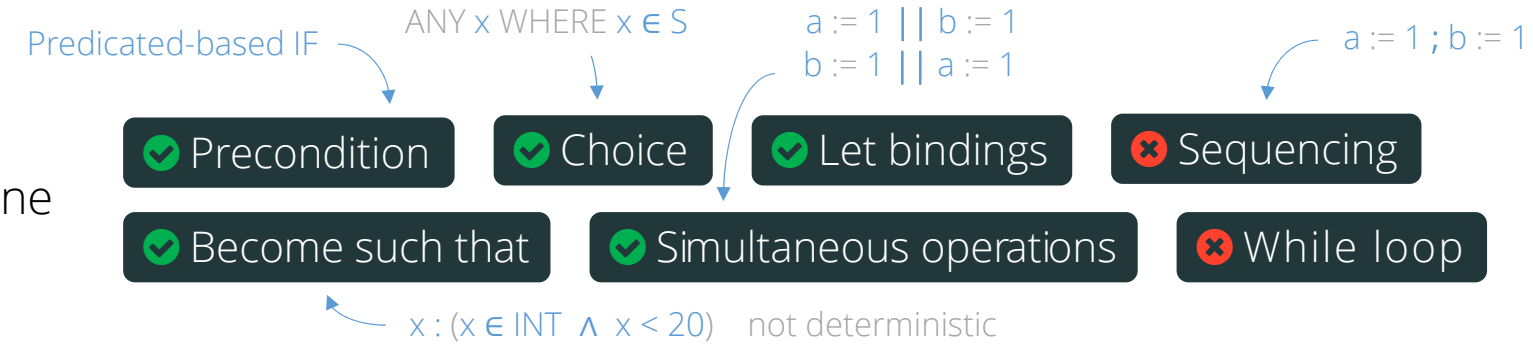
Machine, refinement, implementation

Abstract machine



Refinements

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Machine, refinement, implementation

Abstract machine

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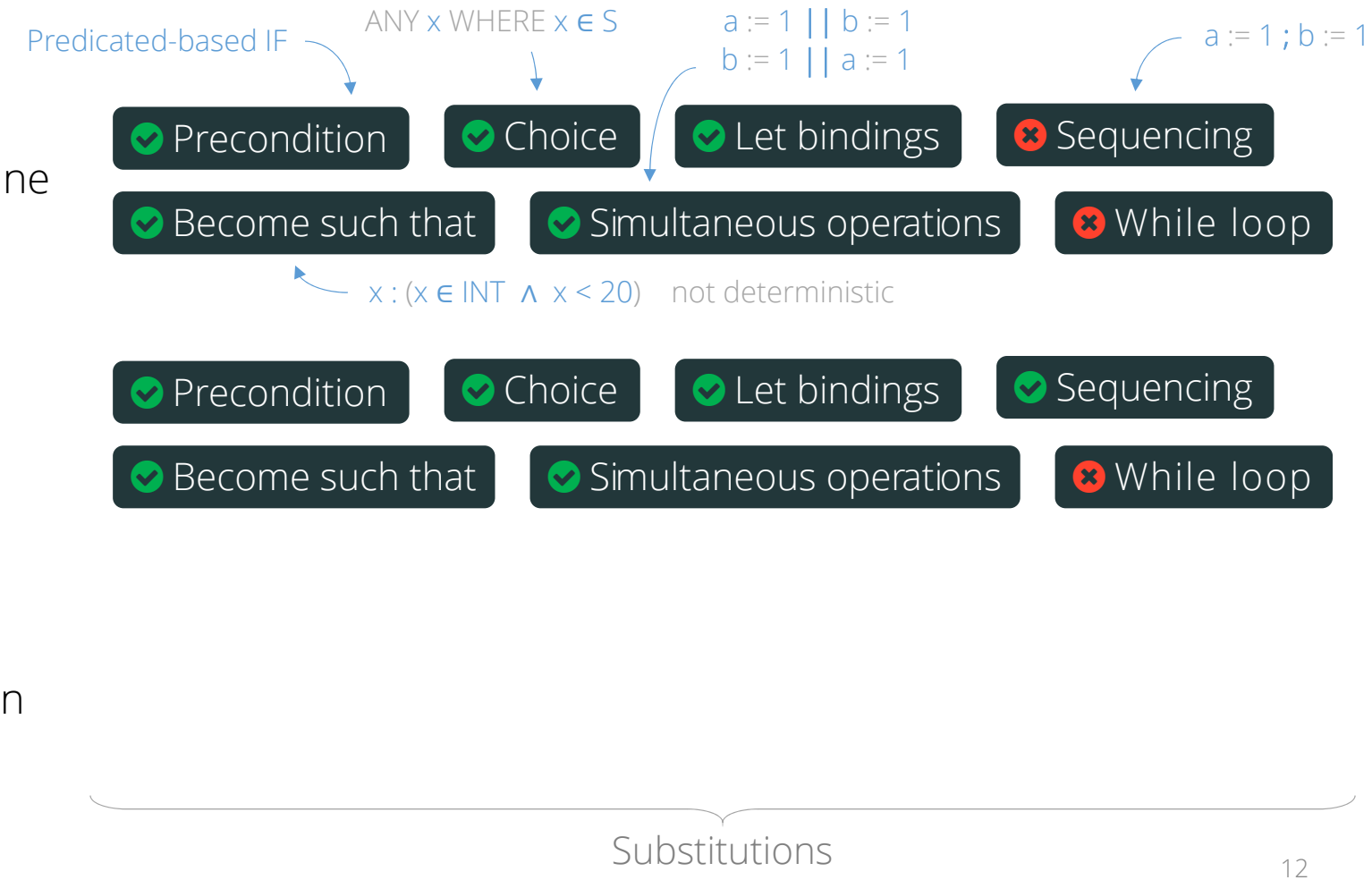
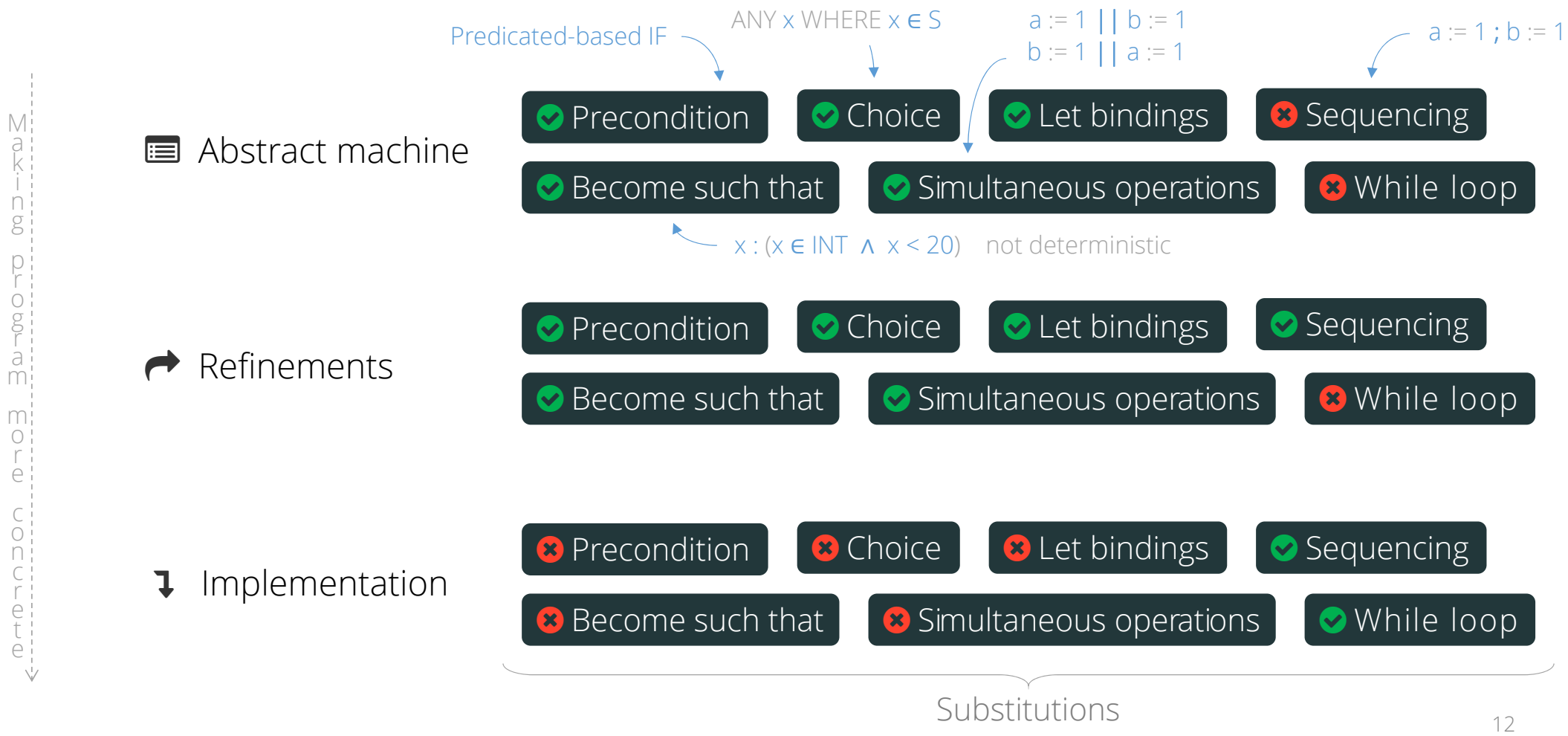


Machine, refinement, implementation

Machine, refinement, implementation



More substitutions

	Machine	Refinement	Implementation
Block	Y	Y	Y
Identical	Y	Y	Y
Becomes Equal	Y	Y	Y
Precondition	Y	Y	N
Assertion	Y	Y	Y
Bounded choice	Y	Y	N
IF conditional	Y	Y	Y
Conditional Bounded choice	Y	Y	N
Case Conditional	Y	Y	Y
Unbounded choice	Y	Y	N
Local Definition	Y	Y	N
Becomes Element of	Y	Y	N
Becomes such that	Y	Y	N
Local Variable	N	Y	Y
Sequencing	N	Y	Y
Operation Call	Y	Y	Y
While Loop	N	N	Y
Simultaneous	Y	Y	N

B language

B language

State oriented

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Hoare logic

$$\{P\}\ C\ \{Q\}$$

B language

State oriented

Hoare logic

$\{P\}\ C\ \{Q\}$

$\{true\}$ y := 3 $\{y = 3\}$

$\{x > 2\}$ x := x*x $\{x > 4\}$

B language

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$$\frac{}{\{P[E/x]\}\ x \coloneqq E\quad \{P\}}\ \text{ASSIGNMENT}$$

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$$\frac{\{P\}\ S\ \{Q\}\,,\ \ \{Q\}\ T\ \{R\}}{\{P\}\ \ S;T\ \ \{R\}}\ \text{COMPOSITION}$$

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$$\frac{\{B \wedge P\}\ S\ \{Q\}\,,\ \ \{\neg B \wedge P\}\ T\ \{Q\}}{\{P\}\ \ \textbf{if}\ B\ \textbf{then}\ S\ \textbf{else}\ T\ \textbf{end}\ \ \{Q\}}\ \text{CONDITIONAL}$$

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$$\frac{\{I \wedge B\}\ S\ \ \{I\}}{\{I\}\ \ \textbf{while}\ B\ \textbf{do}\ S\ \textbf{done}\ \ \{\neg B \wedge I\}}\ \text{WHILE}$$

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Hoare logic

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$$\frac{P\ \rightarrow P',\ \ \{P'\}\ S\ \{Q'\},\ \ Q' \rightarrow Q}{\{P\}\ \ S\ \ \{Q\}}\ \text{CONSEQUENCE}$$

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B language

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Arithmetic

B language

Arithmetic $+ \ - \ \div \ x^y \ \times \ < \ > \ \leq \ \geq$

B language

Arithmetic $+ \quad - \quad \div \quad x^y \quad \times \quad < \quad > \quad \leq \quad \geq \qquad \mathbb{Z} \quad \mathbb{N}$

B language

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B language

Arithmetic $+ \ - \ \div \ x^y \ \times \ < \ > \ \leq \ \geq$ $\mathbb{Z} \ \mathbb{N}$ $\prod \ \sum$

Functions and relations

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Arithmetic $+ \; - \; \div \; x^y \; \times \; < \; > \; \leq \; \geq \qquad \mathbb{Z} \; \mathbb{N} \qquad \prod \; \sum$

Functions and relations

Partial / total functions, surjections, lambda, domain/range manipulations, closure, inversions…

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Sets

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Set comprehension, generalized union & intersections

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Set comprehension, generalized union & intersections

Records

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Trees

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Set comprehension, generalized union & intersections

Records

Trees

Sequences

No algebraic data types

Proofs with B

How does B method handles proofs?

Component
- Abstract machine
- Refinements
- Implementation

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- Refinements
- Implementation

```
MACHINE
    Name(input1, input2, ...)
```

How does B method handles proofs?

Component

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- Refinements
- Implementation

```
MACHINE
   Name(input1, input2, ...)
CONSTRAINTS
   input1 ∈ INT ∧ input2 ∈ INT ...
```

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MACHINE
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CONSTANTS
   cst1, cst2, ...
```

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MACHINE
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CONSTANTS
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VARIABLES
   var1, var2, ...
```

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MACHINE
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CONSTANTS
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VARIABLES
   var1, var2, ...
INVARIANT
   var1 + var2 ∈ {x . X**(1/2) ∈ ℤ} ∧ ...
```

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ASSERTIONS
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lemmas

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Prove all predicates

lemmas

How does B method handles proofs?

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- Refinements
- Implementation

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MACHINE
   Name(input1, input2, ...)
CONSTRAINTS
   input1 ∈ INT ∧ input2 ∈ INT ...
CONSTANTS
   cst1, cst2, ...
VARIABLES
   var1, var2, ...
INVARIANT
   var1 + var2 ∈ {x . X**(1/2) ∈ ℤ} ∧ ...
ASSERTIONS
   predicate1 ∧ predicate2 ∧ ...
INITIALISATION
   var1 := expr || var2 := expr
```

Prove all predicates

lemmas

How does B method handles proofs?

Component
- Abstract machine
- Refinements
- Implementation

```
MACHINE
   Name(input1, input2, ...)
CONSTRAINTS
   input1 ∈ INT ∧ input2 ∈ INT ...
CONSTANTS
   cst1, cst2, ...
VARIABLES
   var1, var2, ...
INVARIANT
   var1 + var2 ∈ {x . X**(1/2) ∈ ℤ} ∧ ...
ASSERTIONS
   predicate1 ∧ predicate2 ∧ ...
INITIALISATION
   var1 := expr || var2 := expr
```

Prove all predicates

For each initialization, prove invariant conservations

lemmas

How does B method handles proofs?

Component
- Abstract machine
- Refinements
- Implementation

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      ...
   varOutput2 ← fun2(i1, i2, ...) =
      ...
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For each initialization, prove invariant conservations

lemmas

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```

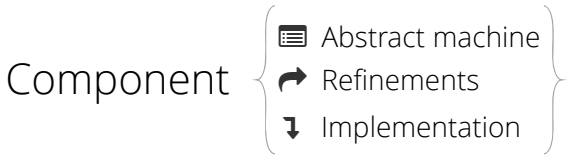
Prove all predicates

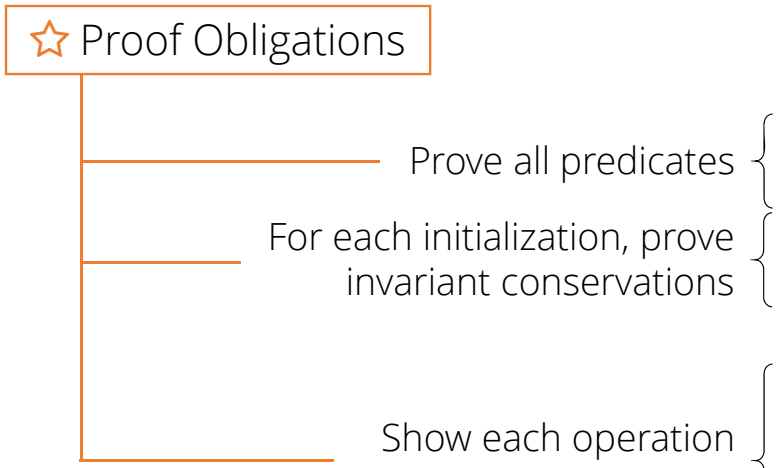
For each initialization, prove invariant conservations

Show each operation conserve invariants

lemmas

How does B method handles proofs?



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```
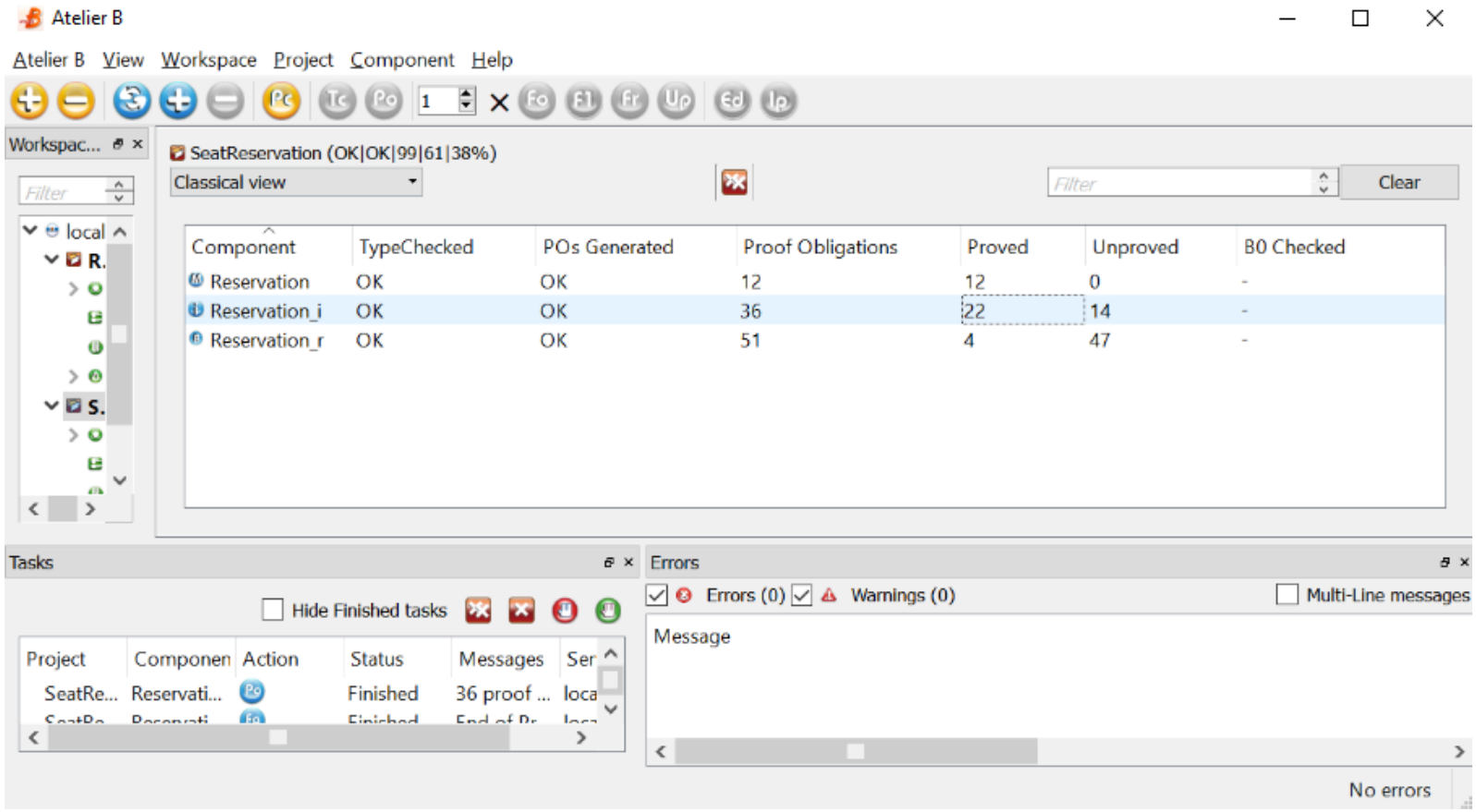
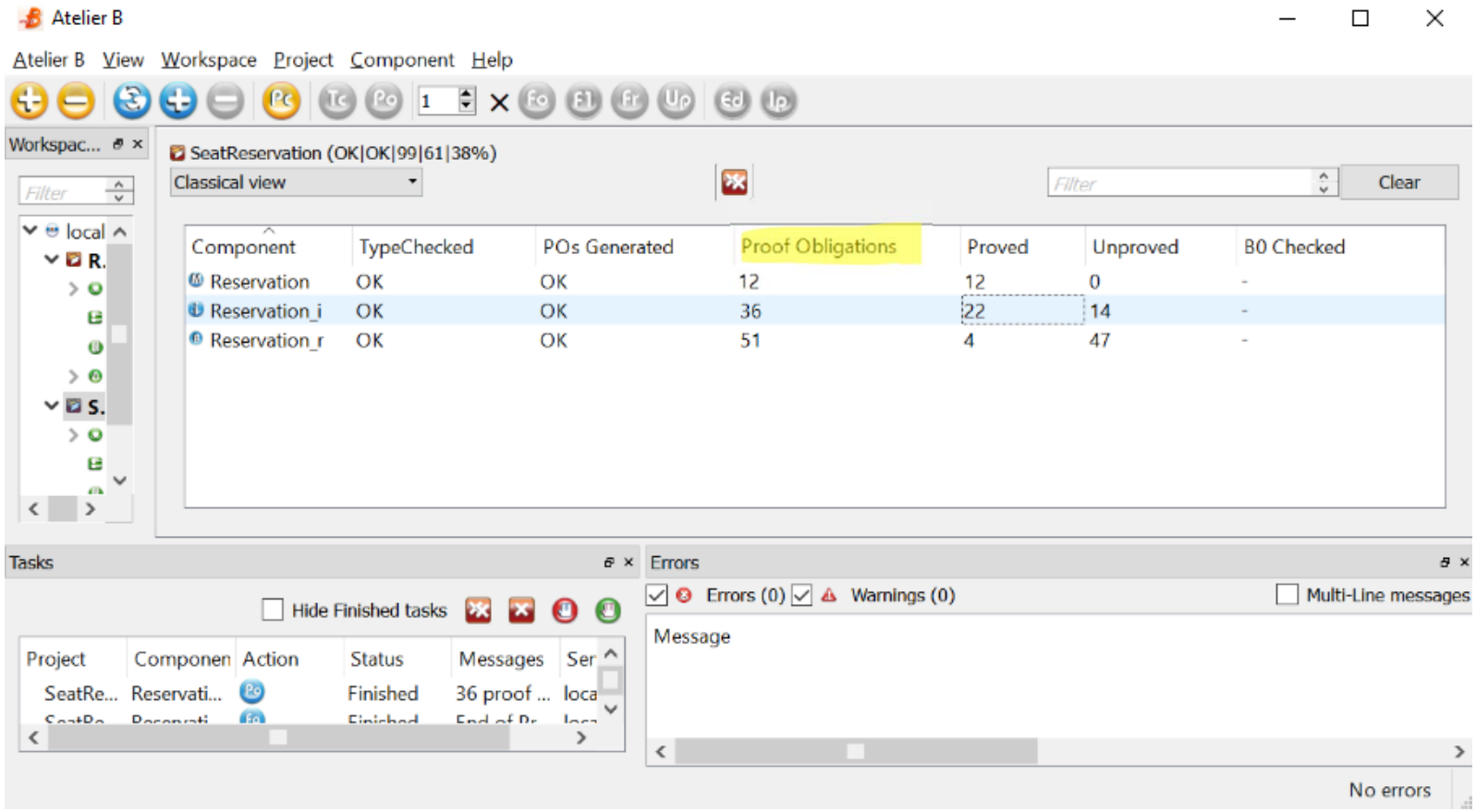

lemmas

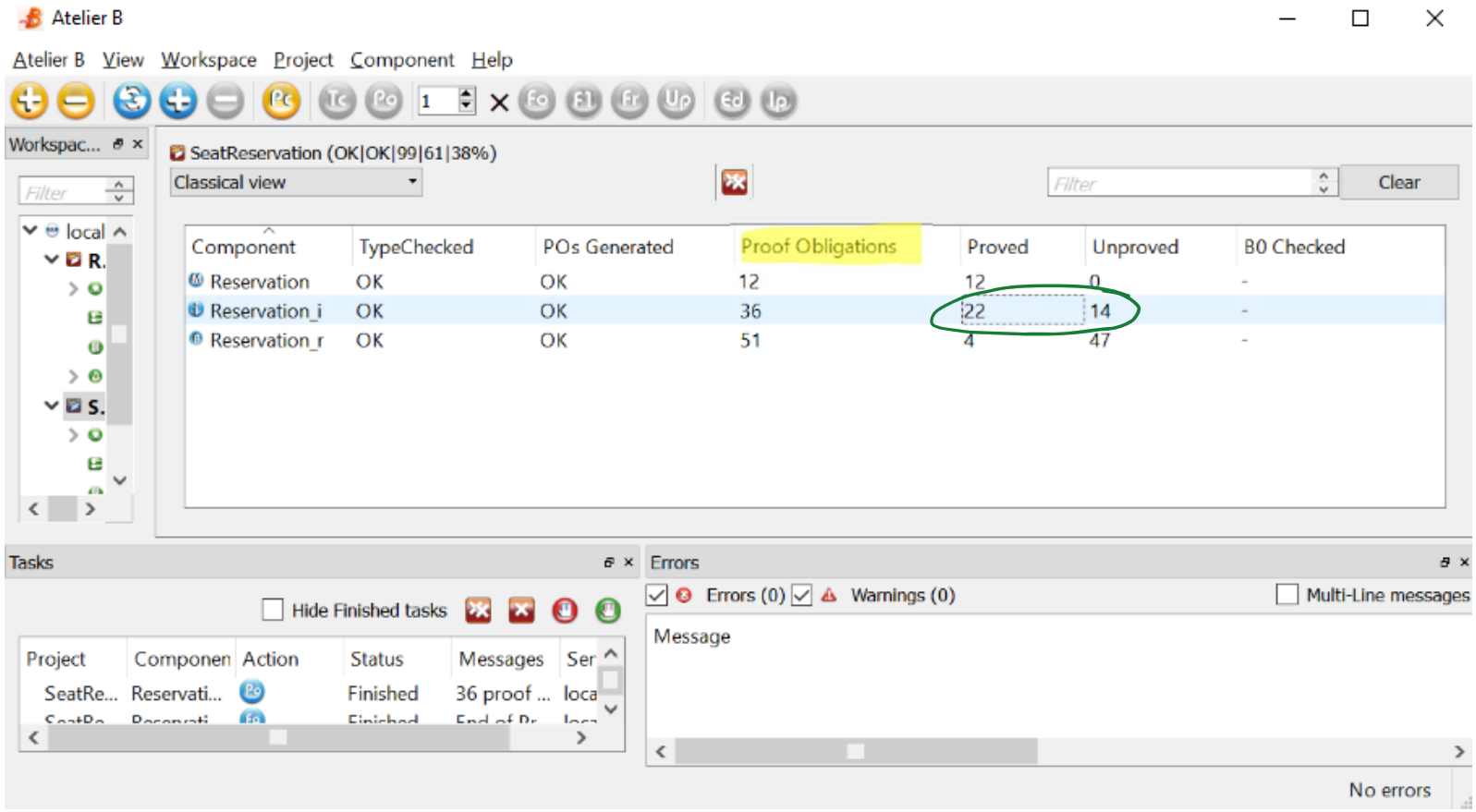
Proving with B method: interactive only



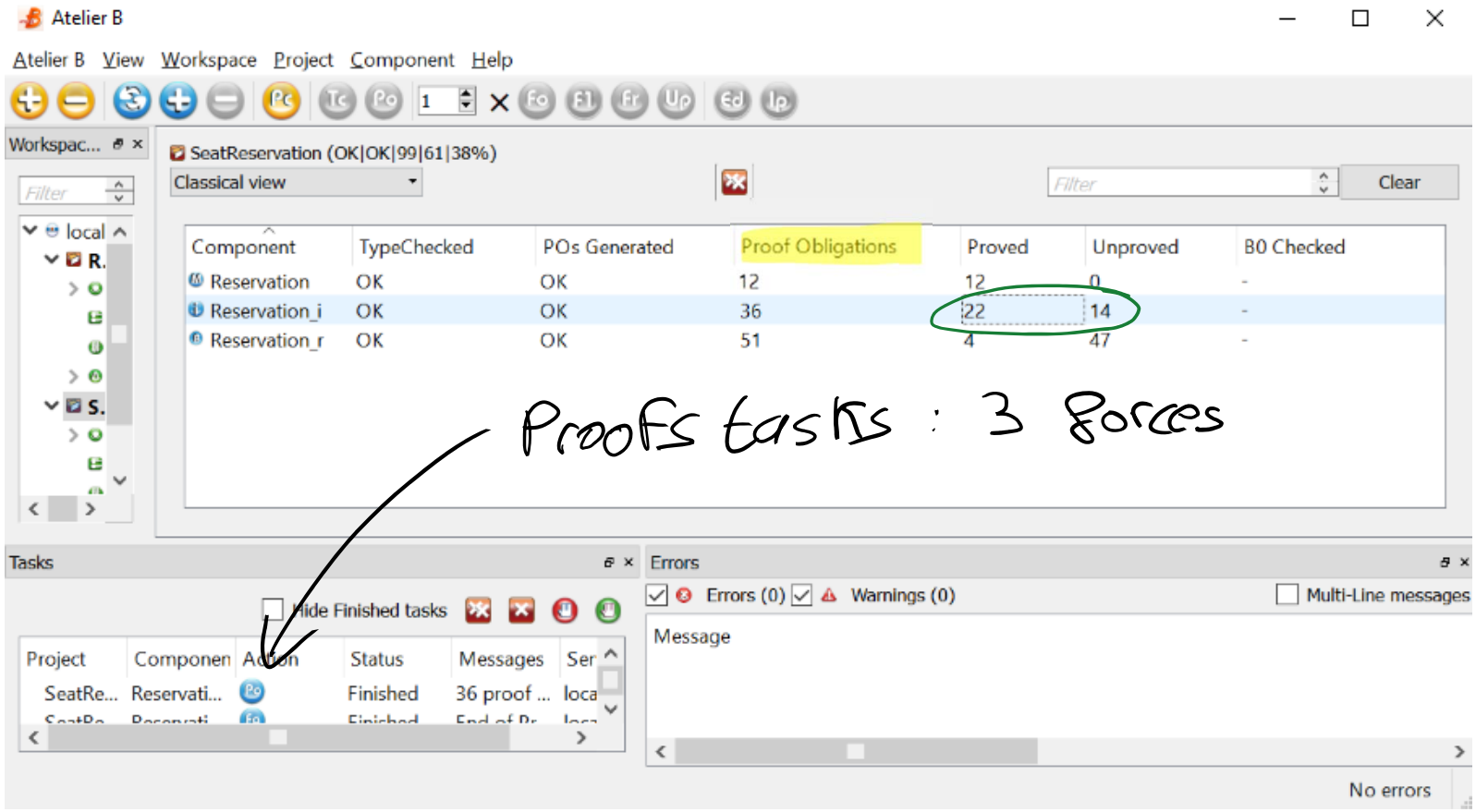
Proving with B method: interactive only



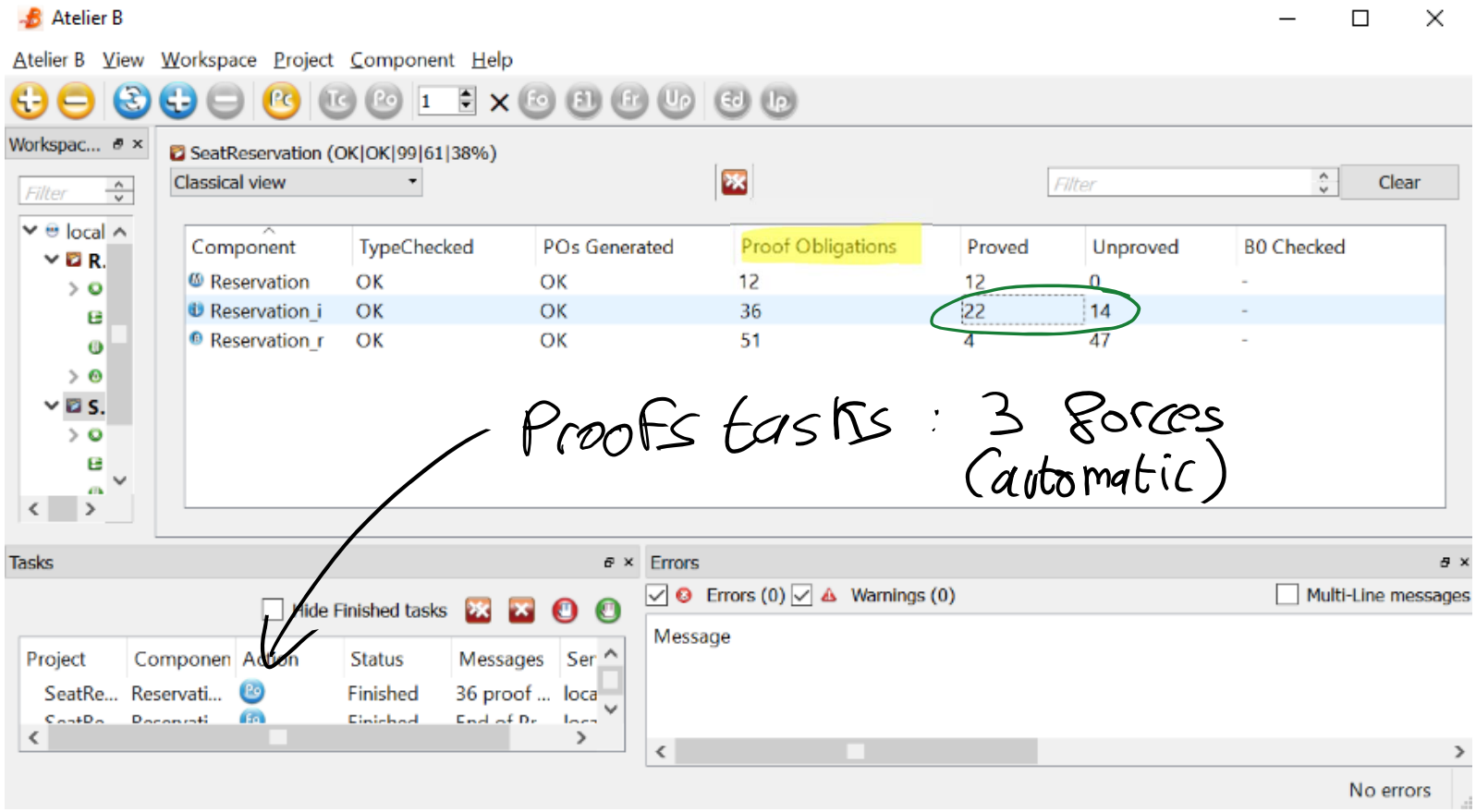
Proving with B method: interactive only



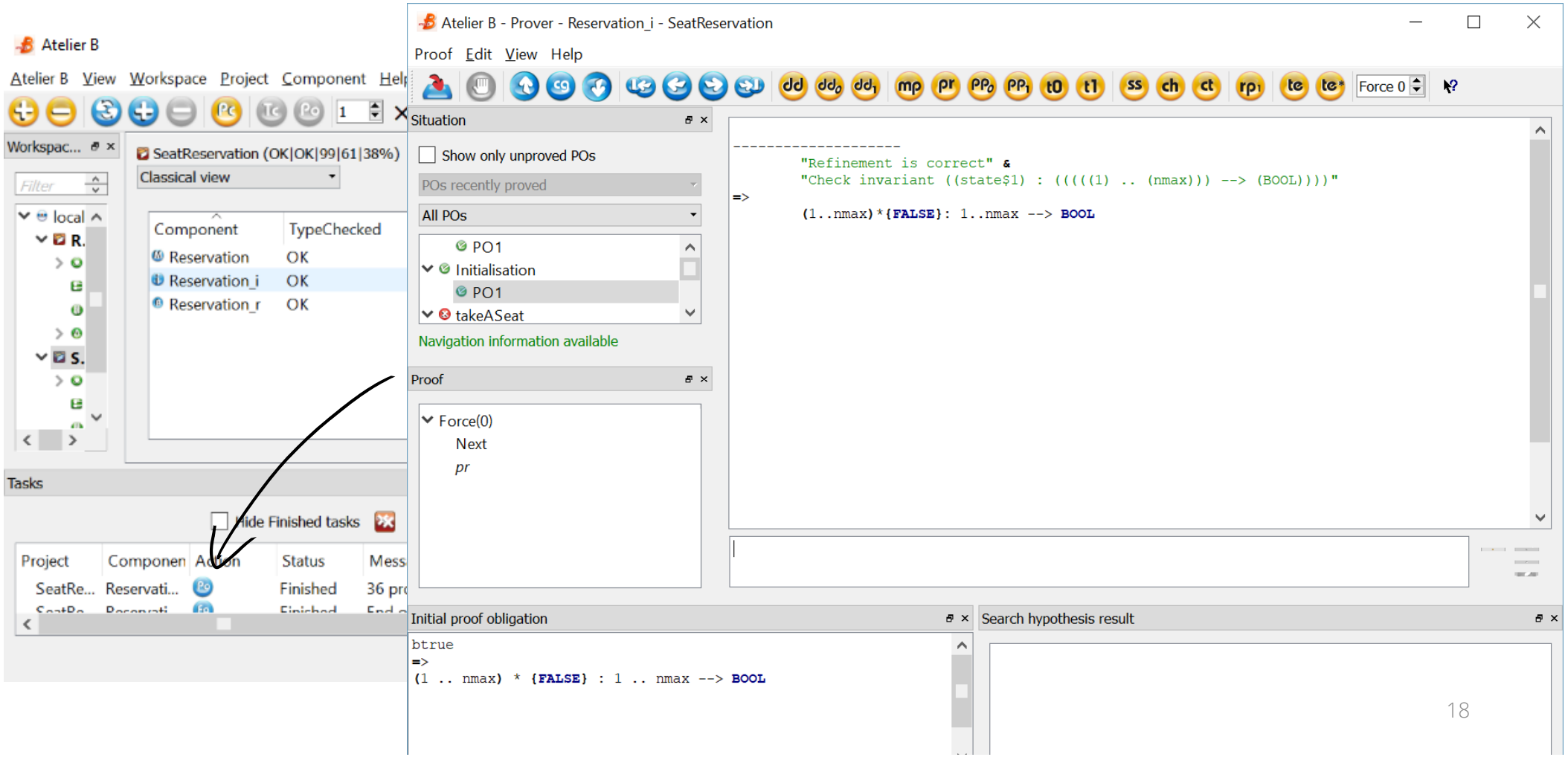
Proving with B method: interactive only



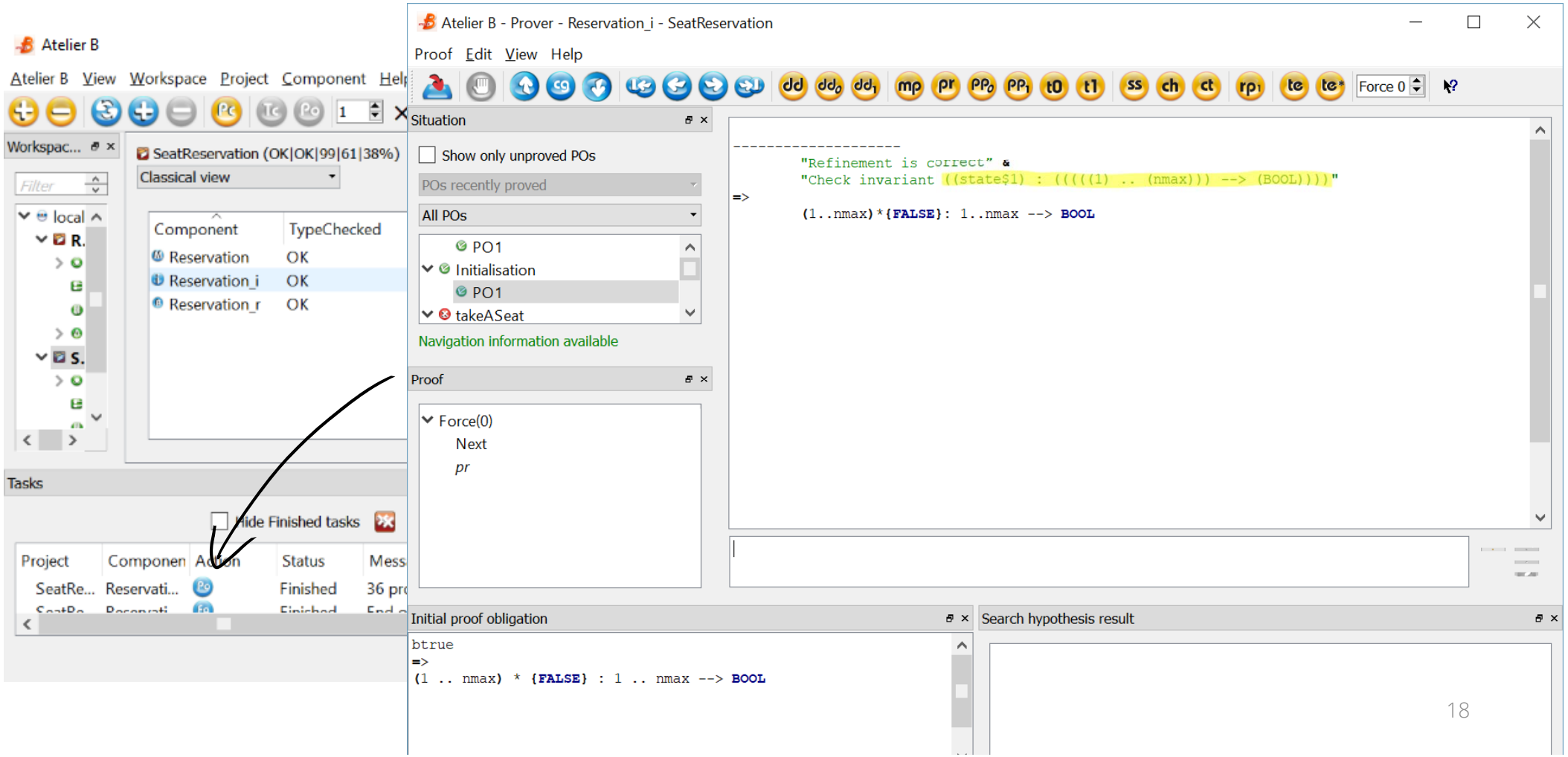
Proving with B method: interactive only



Proving with B method: interactive only



Proving with B method: interactive only



How does B method internally prove things?

First, expressions are normalized

Expression	Normal Form
$n > m$	$m + 1 <= n$
$m < n$	$m + 1 <= n$
$a \ <=> \ b$	$(a => b)\&(b => a)$
$a \ <: \ b$	$a : POW(b)$
$a \ <<: \ b$	$a : POW(b)\&not(a = b)$
$a \ /: \ b$	$not(a : b)$
$a \ /= \ b$	$not(a = b)$
$a \ /<: \ b$	$not(a : POW(b))$
$a \ /<<: \ b$	$a : POW(b) => a = b$
$a : NATURAL$	$a : INTEGER\&0 <= a$
$NATURAL1$	$NATURAL - \{0\}$
$NAT1$	$NAT - \{0\}$
$FIN1(A)$	$FIN(A) - \{\{\}\}$
$POW1(A)$	$POW(A) - \{\{\}\}$
$seq1(A)$	$seq(A) - \{\{\}\}$
$iseq1(A)$	$iseq(A) - \{\{\}\}$
$perm(E)$	$iseq(E)/\backslash(NATURAL - \{0\} + - >> E)$
$<>$	$\{\}$
$\{x, y\}$	$\{x\}\backslash/\{y\}$
$\{x \mid P\}$	$SET(x).P$

How does B method internally prove things?

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$a > b \longrightarrow b + 1 \leq a$

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How does B method internally prove things?

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$a > b \longrightarrow b + 1 \leq a$

$S \subseteq \{1,2,3\} \longrightarrow S \in POW(\{1\} \cup \{2\} \cup \{3\})$

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How does B method internally prove things?

Theory files: list of rules

```
THEORY SimplifyX IS
       (i => (j => k)) == (i & j => k)
;
       (a => bfalse) == not(a)
;
       (bfalse => a) == btrue
;
       (a => btrue) == btrue
;
       (btrue => a) == a
;
       (bvrb(x)) &
       (x\a)
       =>
       #x.a == a
;
       (bvrb(x)) &
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       =>
       #x.(a & b) == (#x.a & b)
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How does B method internally prove things?

Theory files: list of rules

10.000 lines of rules

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User can add rules

⚠ No safe core

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Still, we can prove a rule before adding it

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How does B method internally prove things?

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Still, we can prove a rule before adding it

Automatic prover

- Applies recursively rules
- Case proof
- Tactics

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       (x\b)
```

Interactive proofs

- Proof construction
- Search information
- Browsing proof obligations
- Command repetition

Interactive proofs

Proof construction

Search information

Browsing proof obligations

Command repetition Repeat `(rr)`, loop `(bb)`...

Interactive proofs

Proof construction

Search information

Browsing proof obligations — Back `(ba)`, Reset `(re)`, Next `(ne)`, Previous `(pv)`, Goto `(go)`…

Command repetition — Repeat `(rr)`, loop `(bb)`…

Interactive proofs

Proof construction	
Search information	Search rule / hypothesis / goal, show proof, reduce PO…
Browsing proof obligations	Back `(ba)`, Reset `(re)`, Next `(ne)`, Previous `(pv)`, Goto `(go)`…
Command repetition	Repeat `(rr)`, loop `(bb)`…

Interactive proofs

Proof construction

Interactive proofs

Proof construction

Prover call *automatic proofs & simplification*

Interactive proofs

Proof construction

Prover call *automatic proofs & simplification* **pr**

Interactive proofs

Proof construction

Prover call *automatic proofs & simplification* `pr` `ss`

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr` `ss`
Rule applications		

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr` `ss`
Rule applications		`ar`

Interactive proofs

Proof construction

Prover call *automatic proofs & simplification*	`pr`	`ss`
Rule applications	`ar`	
Rewrite *equality applications, abstract terms*		

Interactive proofs

Proof construction

Prover call *automatic proofs & simplification*	`pr`	`ss`
Rule applications	`ar`	
Rewrite *equality applications, abstract terms*	`eh`	

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`
Rule applications		`ar`	
Rewrite	*equality applications, abstract terms*	`eh`	`ae`

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`
Rule applications		`ar`	
Rewrite	*equality applications, abstract terms*	`eh`	`ae`

$$a + 3 < b \ \wedge \ a + 3 \in S$$
$$\Downarrow$$
$$n < b \ \wedge n \in S$$

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`
Rule applications		`ar`	
Rewrite	*equality applications, abstract terms*	`eh`	`ae`

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`
Rule applications		`ar`	
Rewrite	*equality applications, abstract terms*	`eh`	`ae`
Inference rules	*contradiction, false hypothesis, cases, instantiate*		

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`
Rule applications		`ar`	
Rewrite	*equality applications, abstract terms*	`eh`	`ae`
Inference rules	*contradiction, false hypothesis, cases, instantiate*	`ct`	

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`
Rule applications		`ar`	
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Inference rules	*contradiction, false hypothesis, cases, instantiate*	`ct`	`fh`

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`	
Rule applications		`ar`		
Rewrite	*equality applications, abstract terms*	`eh`	`ae`	
Inference rules	*contradiction, false hypothesis, cases, instantiate*	`ct`	`fh`	`dc`

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`		
Rule applications		`ar`			
Rewrite	*equality applications, abstract terms*	`eh`	`ae`		
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Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`		
Rule applications		`ar`			
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Inference rules	*contradiction, false hypothesis, cases, instantiate*	`ct`	`fh`	`dc`	`se`
Operations on hypothesis	*deduction, add hypothesis…*				

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`		
Rule applications		`ar`			
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Inference rules	*contradiction, false hypothesis, cases, instantiate*	`ct`	`fh`	`dc`	`se`
Operations on hypothesis	*deduction, add hypothesis…*	`dd`			

Interactive proofs

Proof construction

Prover call	*automatic proofs & simplification*	`pr`	`ss`		
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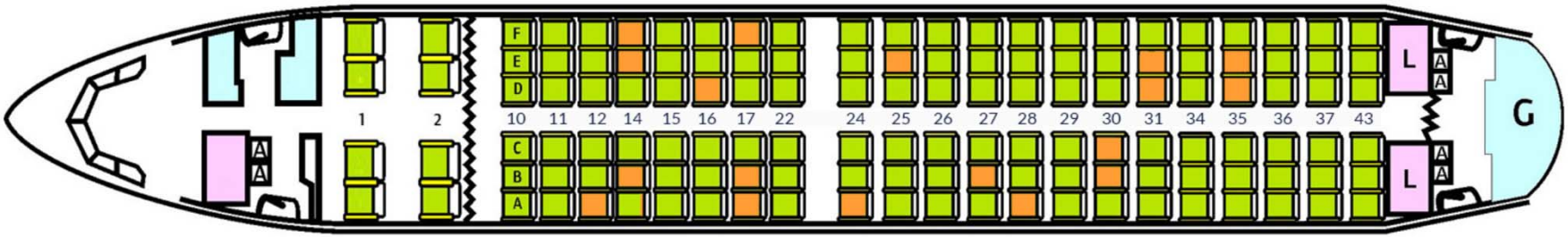
Operations on hypothesis	*deduction, add hypothesis…*	`dd`	`ah`		

Concrete case

Example: seat reservation system



Two operations: *reserve a seat* or *free a seat*

Data : *set of seats* and *sub set of taken seats*

Thank you for listening!

Any questions?