Damas and Milner Call by Value Type Assignments

Presentation Type Theory and Coq

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Contents

Relevant grammars

Terms and values

untyped terms $e ::= v \mid e_1 e_2 \mid \textbf{let}\ x\ \textbf{be}\ e_1\ \textbf{in}\ e_2$

untyped values $v ::= x \mid \lambda x.e \mid \textbf{callcc} \mid \textbf{throw}$

Types and contexts

mono-types $\tau ::= t \mid b \mid \tau_1 \to \tau_2$

poly-types $\sigma ::= \tau \mid \forall t.\sigma$

contexts $\Gamma ::= \bullet \mid \Gamma, x : \sigma$

$\lambda^{\rightarrow}$ typing rules

$$\Gamma \vdash x : \Gamma(x) \; (VAR) \qquad \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x.e : \tau_1 \rightarrow \tau_2} \; (ABS)$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 e_2 : \tau_2} \; (APP)$$

Additional DM+**cont** typing rules

$$\frac{\Gamma \vdash e : \sigma}{\Gamma \vdash e : \forall t.\sigma} \ (t \notin FTV(\Gamma)) \qquad (GEN) \qquad\qquad \frac{\Gamma \vdash e : \forall t.\sigma}{\Gamma \vdash e : [\tau / t]\sigma} \ (INST)$$

$$\Gamma \vdash \mathbf{callcc} : \forall t.(t \text{ cont} \to t) \to t \qquad (CALLCC')$$

$$\Gamma \vdash \mathbf{throw} : \forall s.\forall t.s \text{ cont} \to s \to t \qquad (THROW')$$

Let Typing rule

The following typing rule holds for the **let**-expression

$$\frac{\Gamma \vdash e_1 : \sigma \qquad \Gamma, x : \sigma \vdash e_2 : \tau}{\Gamma \vdash \textbf{let}\ x\ \textbf{be}\ e_1\ \textbf{in}\ e_2 : \tau}\ (x \notin \mathrm{dom}(\Gamma)) \qquad (\textit{POLY-LET})$$

In DM- we restrict **let** expressions so that the bound expression is a value as in CBV.

Call-by-Value CPS Transform

$$
\begin{aligned}
|v|_{cbv} &= \lambda k.k\|v\|_{cbv} \\
|e_1 e_2|_{cbv} &= \lambda k.|e_1|_{cbv}(\lambda x_1.|e_2|_{cbv}(\lambda x_2.x_1 x_2 k)) \\
|\textbf{let } x \textbf{ be } e_1 \textbf{ in } e_2|_{cbv} &= \lambda k.|e_1|_{cbv}(\lambda x.|e_2|_{cbv} k)
\end{aligned}
$$

$$
\begin{aligned}
\|x\|_{cbv} &= x \\
\|\lambda x.e\|_{cbv} &= \lambda x.|e|_{cbv} \\
\|\textbf{callcc}\|_{cbv} &= \lambda f.\lambda k.fkk \\
\|\textbf{throw}\|_{cbv} &= \lambda c.\lambda k.k(\lambda x.\lambda l.cx)
\end{aligned}
$$

Call-by-Value DM- CPS Transform

$$
\begin{aligned}
|v|_{cbv'} &= \lambda k.k||v||_{cbv'} \\
|e_1 e_2|_{cbv'} &= \lambda k.|e_1|_{cbv'}(\lambda x_1.|e_2|_{cbv'}(\lambda x_1.x_1 x_2 k)) \\
|\textbf{let } x \textbf{ be } v \textbf{ in } e|_{cbv'} &= \lambda k.\textbf{let } x \textbf{ be } ||v||_{cbv'} \textbf{ in } (|e|_{cbv'} k)
\end{aligned}
$$

$$
\begin{aligned}
||x||_{cbv'} &= x \\
||\lambda x.e||_{cbv'} &= \lambda x.|e|_{cbv'} \\
||\textbf{callcc}||_{cbv'} &= \lambda f.\lambda k.fkk \\
||\textbf{throw}||_{cbv'} &= \lambda c.\lambda k.k(\lambda x.\lambda l.cx)
\end{aligned}
$$

Call-by-Value Type Transform for DM-

$$
\begin{aligned}
|\tau|_{cbv} &= (||\tau||_{cbv} \to \alpha) \to \alpha \\
|\forall t.\sigma|_{cbv} &= \forall t.|\sigma|_{cbv} \\
\\
||t||_{cbv} &= t \\
||b||_{cbv} &= b \\
||\tau_1 \to \tau_2||_{cbv} &= ||\tau_1||_{cbv} \to |\tau_2|_{cbv} \\
||\forall t.\sigma||_{cbv} &= \forall t.||\sigma||_{cbv}
\end{aligned}
$$

Call-by-Name Type Transform for DM (For Comparisson)

$$
\begin{aligned}
|\tau|_{cbn} &= (||\tau||_{cbn} \to \alpha) \to \alpha \\
|\forall t.\sigma|_{cbn} &= \forall t.|\sigma|_{cbn}
\end{aligned}
$$

$$
\begin{aligned}
||t||_{cbn} &= t \\
||b||_{cbn} &= b \\
||\tau_1 \to \tau_2||_{cbn} &= |\tau_1|_{cbn} \to |\tau_2|_{cbn} \\
||\forall t.\sigma||_{cbn} &= \forall t.||\sigma||_{cbn}
\end{aligned}
$$

Substitution rules

Lemma:

$$||[\tau/t]\sigma||_{cbv} = [||\tau||_{cbv}/t]||\sigma||_{cbv}$$

$$|[\tau/t]\sigma|_{cbv} = [||\tau||_{cbv}/t]\ |\sigma|_{cbv}$$

Proof: by induction on structure of σ.

Meyer-Wand property for DM-

Theorem:

1. If $\Gamma \vdash n : \sigma$ holds in DM-,
   then $|\Gamma|_{cbv} \vdash ||n||_{cbv'} : ||\sigma||_{cbv}$ holds in DM-
2. If $\Gamma \vdash e : \sigma$ holds in DM-,
   then $|\Gamma|_{cbv} \vdash |e|_{cbv'} : |\sigma|_{cbv}$ holds in DM-

Meyer-Wand property for DM-

Theorem:

1. If $\Gamma \vdash n : \sigma$ holds in DM-,
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Proof is practically identical to that of DM with Call-by-Name

An attempt to extend to DM

We can try uing the *cbv* term instead of the specialised *cbv'* transform

We check the same induction step

Counterexample

$$e_0 = \textbf{Let}\ \ f\ \ \textbf{be}\ \ \textbf{callcc}(\lambda k.\lambda x.\textbf{throw} k \lambda y.x)\ \ \textbf{in}\ \ (\lambda x.\lambda y.y)(f 0)(f \textbf{true})$$

This evaluates to 0 while of type bool.

Central Theorem

Theorem: *There is no call-by-value CPS transform* $|e|$ *for* DM *satisfying the following two conditions:*

1. *Equivalence:* $|e|$ *is operationally equivalent to* $|e|_{cbv}$ *under either call-by-value or call-by name semantics.*
2. *Typing:* If with the rules of DM $\Gamma \vdash e : \sigma$ then by the rules of DM+ $||\Gamma||_{cbv} \vdash |e| : |\sigma|_{cbv}$

Thank you for your time