RADBOUD UNIVERSITY NIJMEGEN

EXAM **Formal Languages, Grammars and Automata, NWI-MOL090**

June 25, 2013, 8.30 – 11.30, LIN 5
The maximum number of points per question is given in the margin. The last exercise (7) is "bonus" The grade is obtained by **dividing the number of points by** 10, rounding down to a 10 if needed.

---

1. Consider the following regular expressions

$$r_1 = a(ba)^*b \qquad r_2 = (a(ba)^*b)^* \qquad r_3 = (ab)^*ab \qquad r_4 = (ab)^*$$

(5) (a) Give 2 regular expressions r_i and r_j ($i \neq j$) such that $L(r_i) = L(r_j)$. Show that your answer is correct.

(5) (b) Give 2 regular expressions r_i and r_j such that $L(r_i) \neq L(r_j)$. Show that your answer is correct.

2. Consider the language

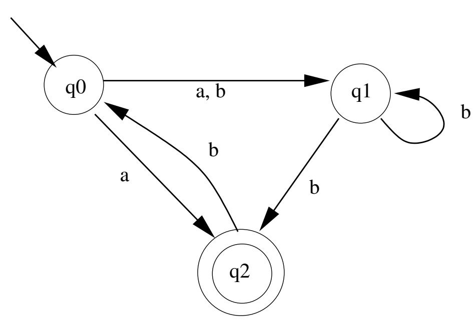
$$L := \{w \in \{a,b\}^* \mid \#_a(w) \text{ is divisible by } 3\}$$

NB "divisible by 3" is in Dutch: "deelbaar door 3"

(6) (a) Give a regular expression e such that $L(e) = L$.

(6) (b) Give a regular grammar G such that $L(G) = L$.

3. Consider the following NFA (non-deteministic finite automaton) M



(6) (a) Indicate for each of the following words whether they are accepted by M: $abba$, $ababa$, $abab$. Explain your answer.

(7) (b) Give a regular expression e such that $L(e) = L(M)$. Explain your answer.

(7) (c) Construct a DFA M' that accepts the same language as M.

Continue on other side

(9) 4. Give a DFA (deterministic finite automaton) that accepts the language L_2 where

$$L_2 = \{w \in \{a,b\}^* \mid \#_a(w) \text{ is even and } w \text{ does not end with } aa\}.$$

5. Consider the language over $\{a,b,c\}$

$$L := \{c\,w\,c\,v\,c \mid w, v \in \{a,b\}^* \text{ and } |w| = |v|\}$$

So, w and v should be of equal length and not contain the symbol c.

(8) (a) Give a context-free grammar G such that $L = L(G)$.

(8) (b) Give a pushdown automaton M such that $L = L(M)$.

(8) (c) Prove that L is not regular.

6. Let M be the PDA with

$$\begin{array}{ll} Q = \{q_0, q_1\} & \delta(q_0, a, \lambda) = \{[q_0, A]\} \\ \Sigma = \{a, b, c\} & \delta(q_0, b, \lambda) = \{[q_0, B]\} \\ \Gamma = \{A, B\} & \delta(q_0, \lambda, A) = \{[q_1, \lambda]\} \\ F = \{q_1\} & \delta(q_1, c, A) = \{[q_1, \lambda]\} \\ & \delta(q_1, \lambda, B) = \{[q_1, \lambda]\} \end{array}$$

(4) (a) Draw a state diagram for M.

(6) (b) Check which of the following words is in $L(M)$ and explain your answer: bba, $abba$ and $baaacc$.

(5) (c) Is $L(b^*a) \subseteq L(M)$? Explain your answer.

(5) (d) For which $p, m, n \in \mathbb{N}$ do we have $b^p a^m c^n \in L(M)$? Explain your answer.

(5) (e) Describe $L(M)$ using set-notation.

(8) 7. [BONUS] Let $\Sigma = \{a, b\}$. Prove that, if L is regular over Σ, then $\mathrm{Max}(L)$ is also regular over Σ, where

$$\mathrm{Max}(L) = \{w \in L \mid \forall v \in \Sigma^* (w\,v \in L \rightarrow v = \lambda)\}$$

So, $\mathrm{Max}(L)$ consists of the words in L that cannot be 'extended'.

END