Huygens College Reflection

Assignment 1, Tuesday, Nov. 10, 2015

1. In the following table, give for every language a word that is in it and a word that is not in it. The alfabet is $\Sigma = \{a, b\}$.

e	$\in \mathcal{L}(e)$	$\notin \mathcal{L}(e)$
aa^*		
$(a \cup b)(a \cup b)^*$		
$(ab^*)^*$		
$a^*(ba^*)^*$		
$ab^*(a \cup \epsilon \cup aa)$		

2. Consider the languages $L_1 = \mathcal{L}((abba)^*)$, $L_2 = \mathcal{L}(a(bba)^*)$, $L_3 = \mathcal{L}((a(bba)^*)^*)$.

   (a) Show that each of these languages is different.

   (b) For which pairs L_i, L_j (with $i \neq j$) do we have $L_i \subseteq L_j$? Prove your answer.

   (c) For which of these languages do we have $L_k\, L_k \neq L_k$? Prove your answer.

3. Give a regular expression for the following languages and explain your answer.

   (a)
   $$\{w \in \{a, b, c\}^* \mid |w| \geq 3\}.$$

   (b)
   $$\{w \in \{a, b\}^* \mid w \text{ begins with } b \text{ and } |w|_b \text{ is even}\}.$$

   (c)
   $$\{w \in \{a, b\}^* \mid bb \text{ doesn't occur in } w\}.$$

4. [This exercise is hard now. They show that the subject is non-trivial. Later we will learn methods to solve this more easily]

   (a) Show that the language
   $$\{w \in \{a, b\}^* \mid aa \text{ occurs exactly twice in } w\}.$$
   is regular.
   [Hint. Beware of the string aaa!]