Huygens College Reflection

Assignment 5, Tuesday, Dec. 15, 2015

Exercise 1

Remember that we can represent a natural number n as the lambda term $\mathbf{c}_n$, its so called *Church numeral*:

$$\mathbf{c}_n \equiv \lambda f a. f^n a,$$

where $f^n a$ denotes n-fold application of f on a. Define

$$A_\times \equiv \lambda n m f a. n(mf)a.$$

(i) Show that $A_\times \mathbf{c}_2 \mathbf{c}_3 = \mathbf{c}_6$;

(ii) Show that $A_\times \mathbf{c}_n \mathbf{c}_m = \mathbf{c}_{n \times m}$.

Exercise 2

Define the map f on trees as follows: $f(t)$ is obtained from t by replacing everywhere l by $j\,l\,(j\,l\,l)$.

(i) Draw the trees t_1, t_2, $f(t_1)$ and $f(t_2)$, with $t_1 := j\,l\,l$, $t_2 := j\,(j\,l\,l)\,l$.

(ii) Give the representation as a λ-term of the trees in (i).

(iii) Construct a λ-term F such that for all trees t

$$F\ulcorner t \urcorner = \ulcorner f(t) \urcorner.$$

Exercise 3

Consider the following context-free grammar

$$S \to c \mid f(S) \mid g(S;S).$$

(i) Which of the following words over $\Sigma := \{f, c, ;, g, (,)\}$ belong to the language?

   (a) c
   (b) f
   (c) g
   (d) $g(f;f)$
   (e) $g(c;f(c))$
   (f) $f(g(c;f))$

(ii) Give the representation as a λ-term of the words in (i) that belong to the language.

(iii) Define λ-terms F and G such that

$$F\ulcorner w \urcorner = \ulcorner f(w) \urcorner$$
$$G\ulcorner w1 \urcorner \ulcorner w2 \urcorner = \ulcorner g(w1;w2) \urcorner$$

Exercise 4

Consider the function h that counts the number of nodes in a tree (where we count a leaf also as a node), so

$$h(j\,l\,l) = 3$$
$$h(j\,l\,(j\,l\,l)) = 5$$

(i) Write down a recursive definition for h, that is fill in

$$h(l) = \ldots$$
$$h(j\,t_1\,t_2) = \ldots h(t_1) \ldots h(t_2) \ldots$$

(ii) Construct a term H that λ-defines h, that is

$$H \ulcorner t \urcorner = \ulcorner h(t) \urcorner$$

for all trees t.