Huygens College Reflection
Assignment 6, Tuesday, Jan. 5, 2016

Exercise 1

(i) Given is the data type `Nat` with $\mathtt{z} : \mathtt{Nat}$, $\mathtt{s} : \mathtt{Nat} \to \mathtt{Nat}$.
Write down the codes (following Böhm, Guerrini, Piperno) of

$$2 = s(sz),\ 3 = s(s(sz)).$$

(ii) Predecessor on `Nat` can be defined recursively:

$$\begin{aligned} p(0) &= 0 \\ p(n+1) &= n. \end{aligned}$$

Using the theory you learned, construct a term P of the form $\langle\langle B_1, B_2\rangle\rangle$, to act on codes of `Nat`, such that

$$\begin{aligned} P(\ulcorner z \urcorner) &= \ulcorner 0 \urcorner \\ P(\ulcorner sn \urcorner) &= \ulcorner n \urcorner. \end{aligned}$$

(iii) Verify $P\ulcorner 3\urcorner = \ulcorner 2\urcorner$.

Exercise 2

(i) Given is `Tree`, the data type with

$$\mathtt{l} : \mathtt{Tree}, \mathtt{j} : \mathtt{Tree}^2 \to \mathtt{Tree}.$$

Write down the codes (following BGP) of

$$t_1 = j\,(j\,l\,l)\,l \quad \text{and} \quad t_2 = j\,l\,(j\,l\,l).$$

(ii) Write down a λ-term $F = \langle\langle D_1, D_2\rangle\rangle$ (to act on codes of `Tree`) such that

$$\begin{aligned} F\ulcorner l \urcorner &= l \\ F\ulcorner j\,t\,s \urcorner &= \ulcorner j\,t\,(j\,t\,s)\urcorner. \end{aligned}$$

(iii) Verify for the F you found that indeed $F\ulcorner j\,l\,(j\,l\,l)\urcorner = \ulcorner j\,l\,(j\,l\,(j\,l\,l))\urcorner$.

Exercise 3

Check the statement on Slide 10 of the course slides, that

$$\begin{aligned} H(\mathtt{Var}\,x) &=_\beta A_1\,x\,H \\ H(\mathtt{App}\,x\,y) &=_\beta A_2\,x\,y\,H \\ H(\mathtt{Abs}\,x) &=_\beta A_3\,x\,H \end{aligned}$$

if we take $H = \langle\langle B_1, B_2, B_3\rangle\rangle$ with

$$\begin{array}{rcl} B_1 & := & \lambda x\, z.A_1 x\langle z\rangle \\ B_2 & := & \lambda x\, y\, z.A_2 xy\langle z\rangle \\ B_3 & := & \lambda x\, z.A_3 x\langle z\rangle. \end{array}$$

and `Var`, `App` and `Abs` as on the slides. (Verify 2 of the equations for H.)

Exercise 4

Show that there is no term F such that

$$F(M\,N) =_\beta N \text{ for all terms } M, N.$$

Exercise 5

Remember the definitions of **true** $:= \lambda xy.x (\equiv \mathbf{K})$ and **false** $= \lambda xy.y (=_\beta \mathbf{KI})$.

(i) Construct a λ-term G such that

$$\begin{array}{rcl} G\ulcorner x\urcorner & = & \mathbf{true} \\ G\ulcorner PQ\urcorner & = & \mathbf{false} \\ G\ulcorner \lambda x.P\urcorner & = & \mathbf{false}. \end{array}$$

(ii) Construct a λ-term V such that

$$\begin{array}{rcl} V\ulcorner x\urcorner & = & \mathbf{true} \\ V\ulcorner PQ\urcorner & = & V\ulcorner P\urcorner \\ V\ulcorner \lambda x.P\urcorner & = & \mathbf{false}. \end{array}$$