Extra exercise Lecture 5

May 10, 2021

Exercise 1. We study the satisfiability om *implicational boolean formulas*, IMP. The formulas of IMP are:

- Atoms, the atoms, $a, b, c, \ldots$,
- Implications, $\varphi \Rightarrow \psi$, for φ, ψ formulas,
- $\bot$.

A valuation $v :$ Atoms $\to \{0, 1\}$ extends in the obvious way to a valuation on all formulas and we say that φ is *satisfiable* in case there is a valuation v for which $v(\varphi) = 1$.

For IMP we also define a type of "normal form", which we call INF (Implicational Normal Form):

- φ is in INF in case it is of the form

$$(c_1 \Rightarrow \ldots \Rightarrow c_n \Rightarrow \bot) \Rightarrow \bot$$

  ($n \geq 0$) where

- c_i are of the form

$$l_1 \Rightarrow l_2 \Rightarrow l_3$$

  where

- each l_i is either an atom a or a negated atom $a \Rightarrow \bot$.

In this notation we have omitted brackets by letting them *associate to the right*, that is: $l_1 \Rightarrow l_2 \Rightarrow l_3$ denotes $l_1 \Rightarrow (l_2 \Rightarrow l_3)$ and similarly $c_1 \Rightarrow \ldots \Rightarrow c_n \Rightarrow \bot$ denotes $(c_1 \Rightarrow \ldots \Rightarrow (c_n \Rightarrow \bot) \ldots)$.

(a) Prove that IMP-SAT, satisifiability of formaulas from IMP, is NP-complete.

(b) Prove that INF-IMP-SAT, satisifiability of formaulas from IMP that are in INF-form, is NP-complete.