qDSA: Small and Secure Digital Signatures with Curve-based Diffie-Hellman Key Pairs

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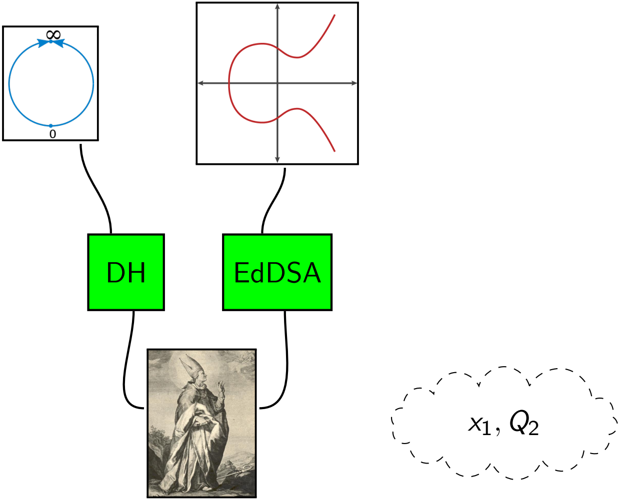
5 December 2017

Curve-based crypto



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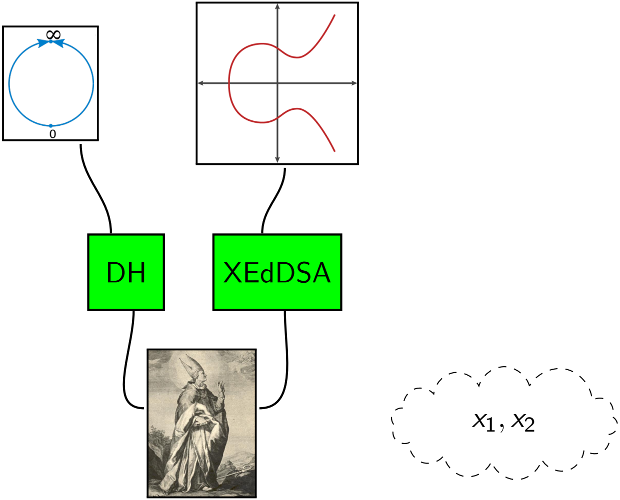

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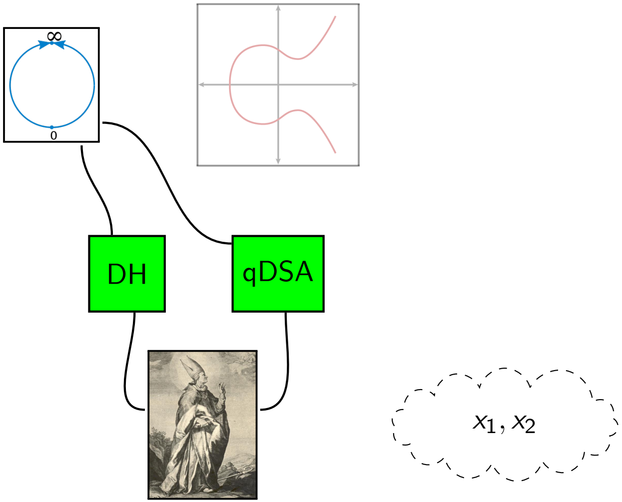

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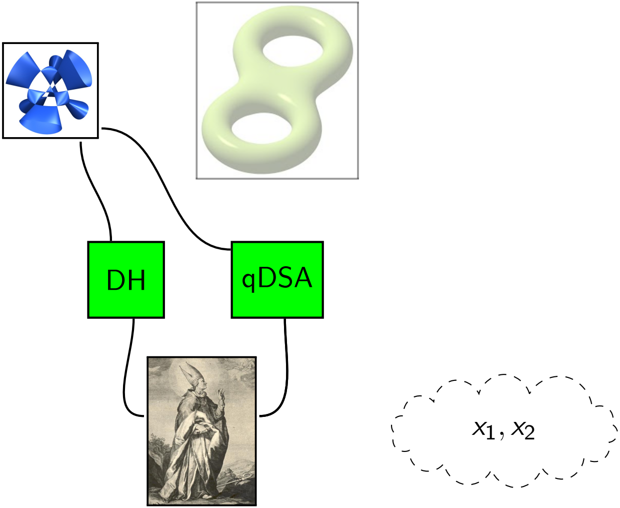
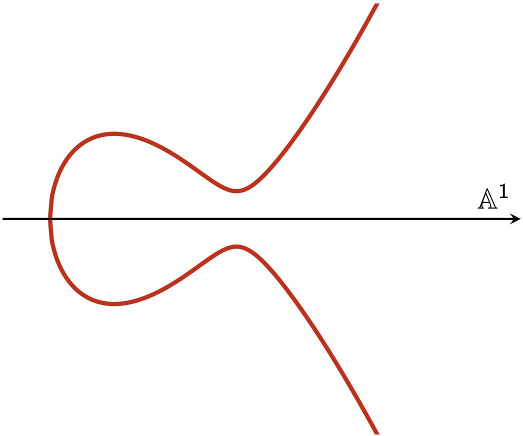


Operations on the Kummer line

Operations on E

(1) $P \mapsto [2]P$

(2) $\{P, Q\} \mapsto P + Q$



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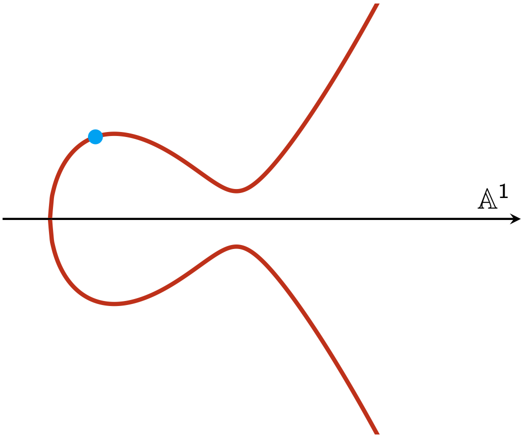

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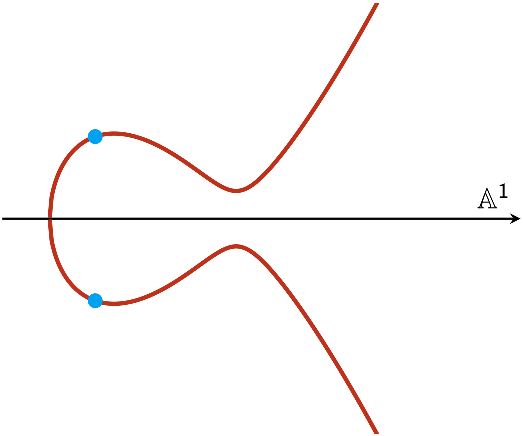

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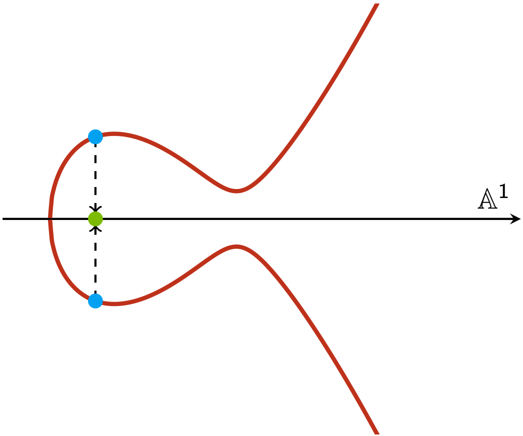

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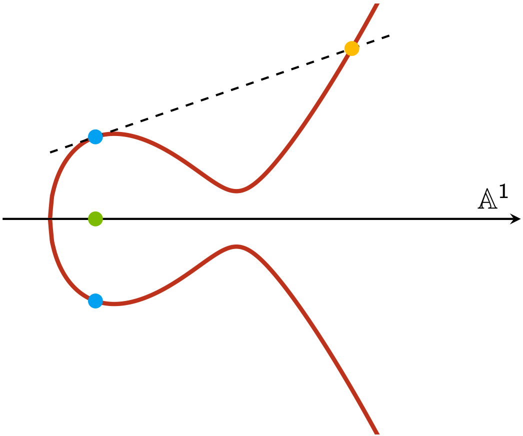

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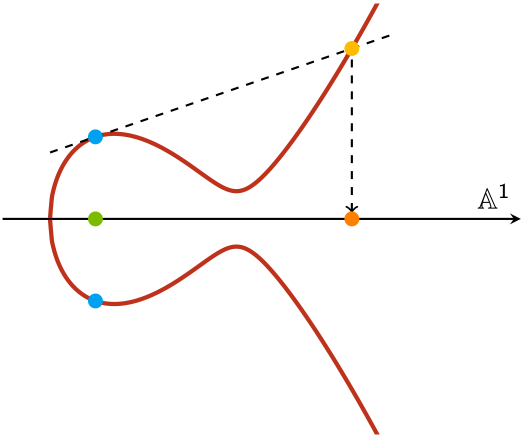

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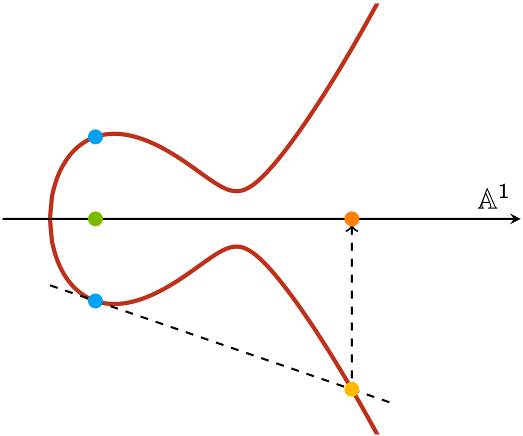

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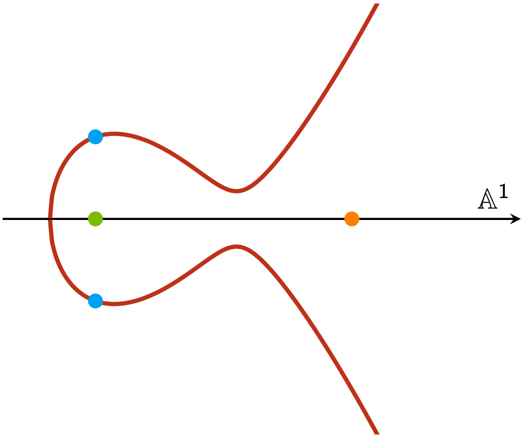

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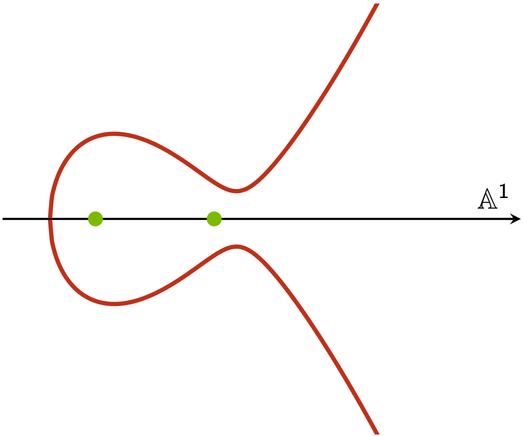

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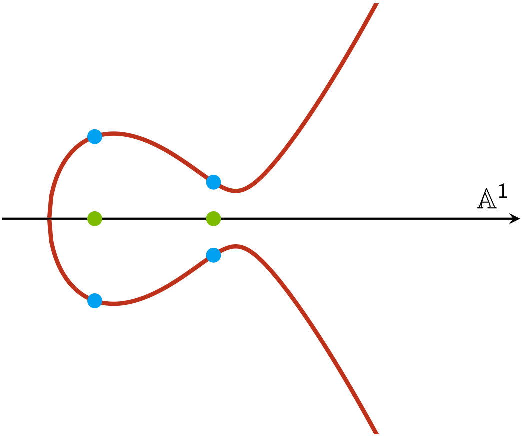

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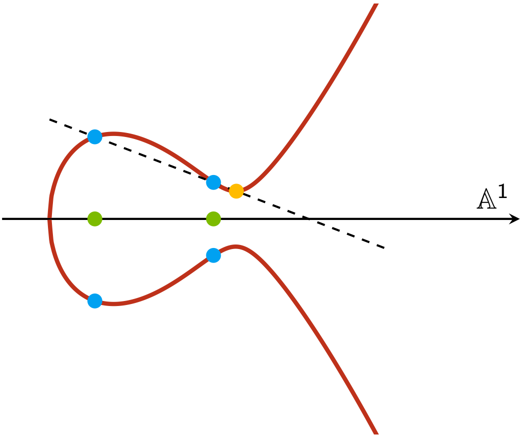

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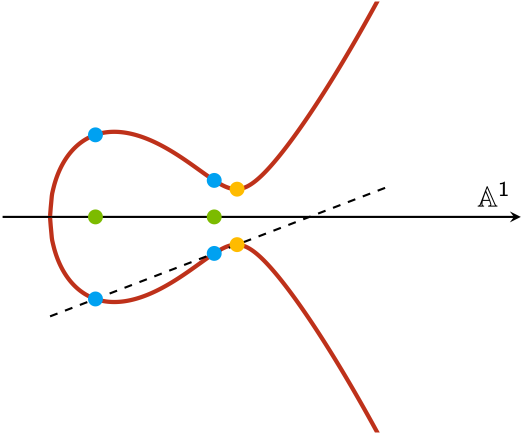

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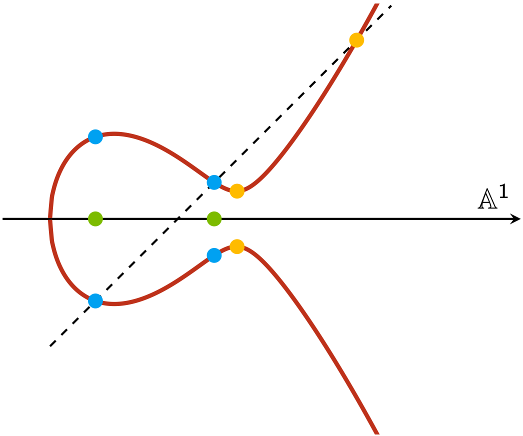

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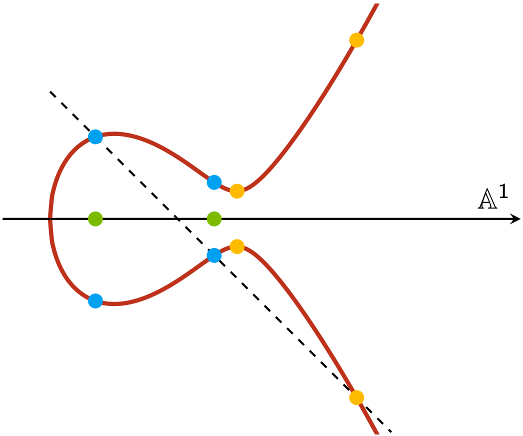

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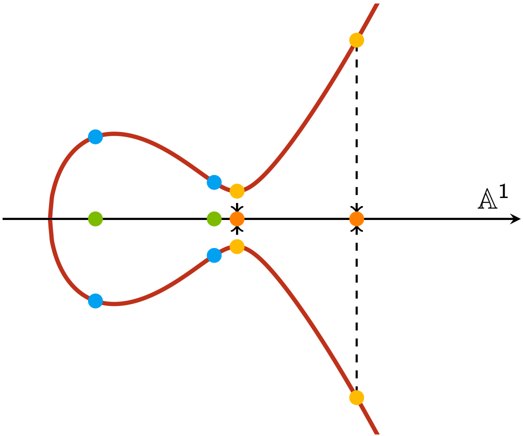

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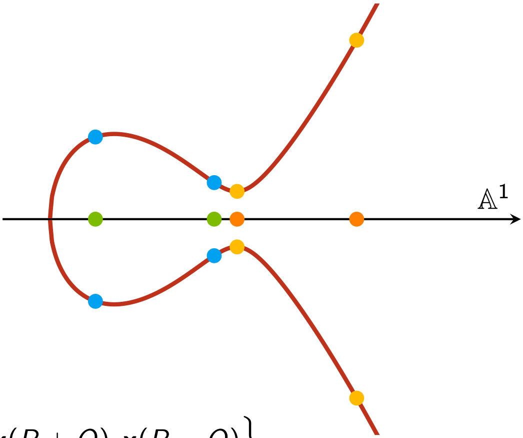

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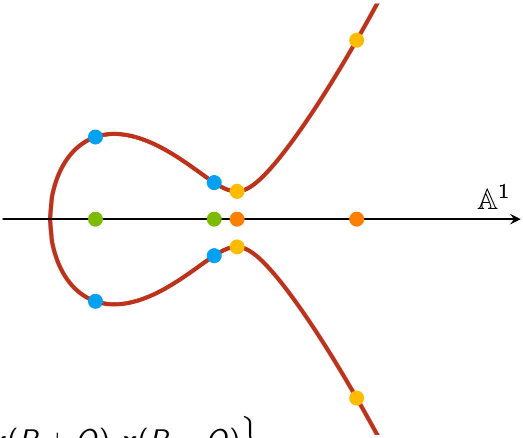

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$\implies \left\{\mathbf{x}(P), \mathbf{x}(Q), \mathbf{x}(P-Q)\right\} \mapsto \mathbf{x}(P+Q)$

Signatures on the Kummer

Starting point: Schnorr signatures [Sch89]

(1) Schnorr identification scheme (group-based)

(2) Apply Fiat-Shamir to make it non-interactive

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Schnorr identification on the quotient (qID)

`Prover`(P, Q, α)	`Comm.`	`Verifier`(P, Q)

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Need $\left\{\mathbf{x}(T_0 + T_1), \mathbf{x}(T_0 - T_1)\right\}$, where $T_0 = [s]P$ and $T_1 = [c]Q$

qSIG and qDSA

$$\underset{\text{(Schn. ID)}}{\text{qID}} \overset{\textit{Fiat-Shamir}}{\Longrightarrow} \underset{\text{(Schn. sig.)}}{\text{qSIG}}$$

qSIG and qDSA

$$\begin{array}{ccccc} \text{qID} & \overset{\textit{Fiat-Shamir}}{\Longrightarrow} & \text{qSIG} & \Longrightarrow & \text{qDSA} \\ \text{(Schn. ID)} & & \text{(Schn. sig.)} & & \text{(EdDSA)} \end{array}$$

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(5) **Side-channels & faults.** Add countermeasures *depending on context* of implementation

Biquadratic forms on $\mathbb{P}^1$ for Montgomery curves

$$\Big\{\mathbf{x}(P), \mathbf{x}(Q)\Big\} \mapsto \Big\{\mathbf{x}(P+Q), \mathbf{x}(P-Q)\Big\}$$

Biquadratic forms on $\mathbb{P}^1$ for Montgomery curves

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$$\begin{aligned} X_3 Z_4 + X_4 Z_3 &= B_{10}\,, \quad B_{10} = \nu \cdot \big[\,(X_1 Z_2 - X_2 Z_1)(X_1 Z_2 + X_2 Z_1) \\ &\qquad + 2 A X_1 X_2 Z_1 Z_2\big] \end{aligned}$$

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$$\left.\begin{aligned} X_3X_4 &= B_{00}\,, & B_{00} &= \nu \cdot (X_1X_2 - Z_1Z_2)^2 \\ Z_3Z_4 &= B_{11}\,, & B_{11} &= \nu \cdot (X_1Z_2 - X_2Z_1)^2 \end{aligned}\right\} \textbf{xADD}$$

$$X_3Z_4 + X_4Z_3 = B_{10}\,, \quad B_{10} = \nu \cdot \left[(X_1Z_2 - X_2Z_1)(X_1Z_2 + X_2Z_1) + 2AX_1X_2Z_1Z_2 \right]$$

$$\begin{pmatrix} X_3X_4 & * \\ X_3Z_4 + X_4Z_3 & Z_3Z_4 \end{pmatrix} = \nu \cdot \begin{pmatrix} B_{00} & * \\ B_{10} & B_{11} \end{pmatrix} \qquad \mathbf{(4 \times 4)}$$

Thus $(X_3 : Z_3)$ and $(X_4 : Z_4)$ are the **unique** solutions to

$$B_{11}X^2 - 2 \cdot B_{10}XZ + B_{00}Z^2 = 0 \qquad \textbf{(6 eqns)}$$

Cost of computing biquadratic forms

g	**Func.**	**M**	**S**	**C**
1	`Check`	8	3	1
	`Ladder`	1 280	1 024	256
2	`Check`	76	8	88
	`Ladder`	1 799	3 072	3 072

Table: Cost of B_{IJ}

Implementing the scheme (at 128-bit security)

g.	Ref.	Object.	Function.	CC.	Stack.
	This	`Curve25519`	`sign`	14 M	512 B
1	[NLD15]	`Ed25519`	`sign`	19 M	1 473 B
	[Liu+17]	Four$\mathbb{Q}$	`sign`	5 M	1 572 B

Table: AVR ATmega comparison (rounded)

Implementing the scheme (at 128-bit security)

g.	Ref.	Object.	Function.	CC.	Stack.
	This	`Curve25519`	`verify`	25 M	644 B
1	[NLD15]	`Ed25519`	`verify`	31 M	1 226 B
	[Liu+17]	Four$\mathbb{Q}$	`verify`	11 M	4 957 B

Table: AVR ATmega comparison (rounded)

Implementing the scheme (at 128-bit security)

g.	Ref.	Object.	Function.	CC.	Stack.
2	**This**	`Gaudry-Schost`	`sign`	10 M	417 B
	[Ren+16]	`Gaudry-Schost`	`sign`	10 M	926 B

Table: AVR ATmega comparison (rounded)

Implementing the scheme (at 128-bit security)

g.	Ref.	Object.	Function.	CC.	Stack.
2	**This**	`Gaudry-Schost`	`verify`	20 M	609 B
	[Ren+16]	`Gaudry-Schost`	`verify`	16 M	992 B

Table: AVR ATmega comparison (rounded)

Thanks for your attention!

`http://www.cs.ru.nl/~jrenes/`

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