Program verification with Why3

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February 7, 2019

Preliminaries

The first part of this course is on Why3

- Main sessions: thursdays, 8:30 HG00.068

- Tutorial hour: next wednesday, 10:30 MERC I 00.28
  Slot will not always be used – check announcements!

Useful if you remember something from:

- Mathematical Structures
- Assertion and Argumentation
- Semantics and Correctness
- Functional Programming

Practicalities

Reference materials
Why3 Tutorial by J.C. Filliâtre:

`why3.lri.fr/vtsa-18/notes-why3.pdf`

Manual:

`why3.lri.fr/manual.pdf`

Exercises: see Brightspace

- Important you do these!
- **Work in pairs!**
- Deadline: tuesday 12:00

Help? Contact: `m.schoolderman@cs.ru.nl`

Motivation

Formal verification

Validation
Writing the correct program

- Are informal requirements captured in a *specification*?

Verification
Writing the program correctly

- Does the program match the specification?

Formal verification

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Verification
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- Does the program match the specification?

Formal verification
The art of using rigorous, mathematical techniques for verification

- Prove that a program matches a **formal specification**!

The dawn of computing science

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	… **crickets** …
1967	Floyd: “Assigning Meanings to Programs”
1969	Hoare logic (Axiomatic semantics)
1975	Dijkstra: weakest preconditions

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… fast forward …

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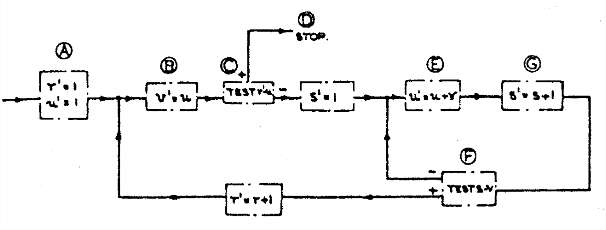
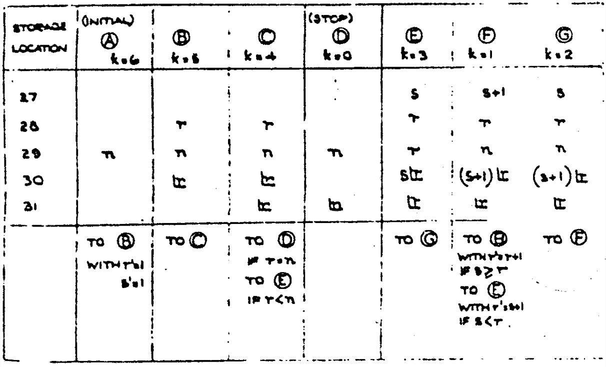
… fast forward …

2019 Formal verification seldomly used

Turing's notes





STORAGE LOCATION	(INITIAL) Ⓐ $k=6$	Ⓑ $k=5$	Ⓒ $k=4$	(STOP) Ⓓ $k=0$	Ⓔ $k=3$	Ⓕ $k=1$	Ⓖ $k=2$
27					s	$s+1$	s
28		r	r		r	r	r
29	n	n	n	n	r	n	n
30		$\underline{\lvert r}$	$\underline{\lvert r}$		$s\underline{\lvert r}$	$(s+1)\underline{\lvert r}$	$(s+1)\underline{\lvert r}$
31			$\underline{\lvert r}$	$\underline{\lvert n}$	$\underline{\lvert r}$	$\underline{\lvert r}$	$\underline{\lvert r}$
	TO Ⓑ WITH $r'=1$ $u'=1$	TO Ⓒ	TO Ⓓ IF $r=n$ TO Ⓔ IF $r<n$		TO Ⓖ	TO Ⓑ WITH $r'=r+1$ IF $s\geq r$ TO Ⓔ WITH $s'=s+1$ IF $s<r$.	TO Ⓕ

Critical software bugs

Testing shows the presence, not the absence of bugs – Dijkstra

1996	Ariane 5: uncaught runtime exception
1999	NASA: confused imperial and metric system
2008	Debian OpenSSL: RNG seeded with well-known data
2009	Toyota "unintended acceleration": stack overflow
2014	Apple SSL: `goto fail`
2014	HeartBleed: buffer overflow
2014	ShellShock: Incorrect input processing (undetected for 25 years)

More stories:

`http://www.cs.tau.ac.il/~nachumd/horror.html`

Bugs in the Java JDK

2006 “Nearly All Binary Searches and Mergesorts are Broken” – Joshua Bloch:[¶]

```
int mid = (low+high)/2
int midVal = a[mid]
```

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Will cause overflow if `low+high`$\geq 2^{31}$!

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Bugs in the Java JDK

2006 “Nearly All Binary Searches and Mergesorts are Broken” – Joshua Bloch:[¶]

```
int mid = (low+high)/2
int midVal = a[mid]
```

Will cause overflow if `low+high`$\geq 2^{31}$! Fixing this in C:

```
int mid = low + (high−low)/2
```

[¶] `https://ai.googleblog.com/2006/06/extra-extra-read-all-about-it-nearly.html`

What can we do about this?

Proofs can show the absence of (certain) bugs.

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The Bad News
Formal proofs are *hard*, *tedious*, *time-consuming*, and *error-prone*.

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The Bad News
Formal proofs are *hard*, *tedious*, *time-consuming*, and *error-prone*.

The Good News
Major advances in *computional power* and *artificial intelligence*

- Interactive theorem provers: Coq, PVS, Isabelle/HOL
- Fully automated provers: Z3, CVC4, E, Alt-Ergo, ...

Another reason to use machine intelligence

Do you trust your own proofs?

Do you trust other people's proofs?

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Cryptanalysis of OCB2 **– Inoue & Minematsu**

- OCB2: authenticated encryption, 'proven secure' in 2004
- Broken in 2018?!
  `https://eprint.iacr.org/2018/1040.pdf`

Some high-profile stories

AMD K5	Verification of `fdiv` using ACL2 (1995)
Paris Métro	Driverless *Ligne 14* verified using B-Method (1998)
Hyper-V	Hypervisor verified using VCC and Z3 (2005)
CompCert	C compiler verified using Coq (2009)
seL4	micro-kernel verified using Isabelle/HOL (2009)

Work in progress:

Project Everest	Verified HTTPS stack using F*
CakeML	Bootstrapping, verified compiler for ML

The state of the art

Modern systems for program verification

- Why3 (INRIA)
- F* (Microsoft Research)
- Frama-C/WP (INRIA+CEA)

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Common complaint from industry:
"Give us a system that we can actually use"

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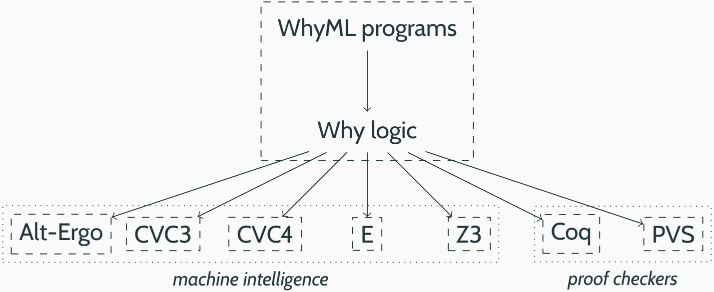
Common complaint from industry:
“Give us a system that we can actually use”

Let’s see how far we get with Why3

- Recently had major updates
- Expertise present at Radboud

The Why3 Platform

specifications + programs ↦ verification conditions


WhyML programs

Why logic

Alt-Ergo

CVC3

CVC4

E

Z3

Coq

PVS

machine intelligence

proof checkers

Layers of Why3

WhyML consists of two parts:

1. A pure logic system
    - Usage: theorem proving.
2. A programming language
    - Usage: modelling programs, intermediate language

Three ways of using Why3

❶ A pure logic system

- First order logic + pure functions (no side effects)
- Proofs discharged by *automatic provers*
- Why3 keeps track of dependencies between proofs
- Ability to produce (potential) *counter-examples*

Why3 logic

```
module Example

use int.Int

predicate odd (x: int) = exists k: int. x = 2*k+1

function sqr (x: int): int = x*x

lemma odd_square:
  forall x: int. odd x -> odd (sqr x)

end
```

WhyML

❷ WhyML as a programming language

- Imperative programming (`while` loops, mutable data)
- Function contracts: pre- and postconditions
- Algebraic data types with pattern matching
- Type inference (like Haskell, ML)
- Control-flow: `break`, `continue`, `return`
- Why3 generates verification conditions

WhyML programs

```
let foo (x: int): int
  requires { x >= 0 }
  ensures { result >= 0 }
= let z = ref 0 in
  let odd = ref 1 in
  let sum = ref 1 in
  while !sum <= x do
    z := !z + 1;
    odd := !odd + 2;
    sum := !sum + !odd;
  done;
  return !z
```

Theoretical background

Hoare logic

Pre- and postconditions: $\{P\}$ **S** $\{Q\}$

- *Partial correctness*: if P holds, and we run **S**, then Q holds when it terminates.
- **Note**: Doesn't say **S** will actually terminate!

Comes with derivation rules:

```
{ x = 42 }
{ x+1 = 43 }
y := x + 1
{ y = 43 }
```

$$\{P[x \mapsto a]\}\mathbf{x{:=}a}\{P\}$$

$$\frac{\{P\}\mathbf{S}_1\{Q\} \quad \{Q\}\mathbf{S}_2\{R\}}{\{P\}\mathbf{S}_1;\mathbf{S}_2\{R\}}$$

Do you enjoy Hoare logic?

```
{ x >= 0 }
while x > 0 do
  x := x-1
done
{ x = 0 }
```

Do you enjoy Hoare logic?

```
{ x >= 0 }

while x > 0 do


  x := x-1


done

{ x = 0 }
```

Do you enjoy Hoare logic?

```
{ x >= 0 }
{ INV }
while x > 0 do
  { x > 0 /\ INV }

  x := x-1

  { INV }
done
{ not (x > 0) /\ INV }
{ x = 0 }
```

Do you enjoy Hoare logic?

```
{ x >= 0 }
{ x >= 0 }
while x > 0 do
  { x > 0 /\ x >= 0 }

  x := x-1

  { x >= 0 }
done
{ not (x > 0) /\ x >= 0 }
{ x = 0 }
```

Do you enjoy Hoare logic?

```
{ x >= 0 }
{ x >= 0 }
while x > 0 do
  { x > 0 /\ x >= 0 }
  { x-1 >= 0 }
  x := x-1
  { x >= 0 }
  { x >= 0 }
done
{ not (x > 0) /\ x >= 0 }
{ x = 0 }
```

Some observations

Hoare logic proofs are mostly mechanical, except:

- Finding loop invariants
- Proving that one condition follows from another

And can only show *partial correctness*!

Weakest liberal precondition

A predicate $wlp(S, Q)$, so that $\{wlp(S, Q)\}$ **S** $\{Q\}$

- Instead of deriving $\{P\}$ **S** $\{Q\}$, just show $P \rightarrow wlp(S, Q)$

Weakest liberal precondition

A predicate $wlp(S, Q)$, so that $\{wlp(S, Q)\}$ **S** $\{Q\}$

- Instead of deriving $\{P\}$ **S** $\{Q\}$, just show $P \rightarrow wlp(S, Q)$

$$\begin{aligned}
wlp(\texttt{x:=e}, Q) &= Q[x \mapsto e] \\
wlp(\texttt{e1;e2}, Q) &= wlp(e1, wlp(e2, Q)) \\
wlp(\texttt{if b then e1 else e2}, Q) &= (b \rightarrow wlp(e1, Q)) \land (\neg b \rightarrow wlp(e2, Q)) \\
wlp(\texttt{while b do S}, Q) &= INV \land \\
&\quad \forall v \in S.INV \rightarrow (b \rightarrow wlp(S, INV)) \land (\neg b \rightarrow Q)
\end{aligned}$$

wlp: *Computation*

$$
\begin{aligned}
wlp(\textbf{while...done}, x = 0) = {} & INV \wedge \\
& \forall x. INV \rightarrow (x > 0 \rightarrow wlp(\textbf{x:=x-1}, INV)) \wedge \\
& (\neg(x > 0) \rightarrow x = 0)
\end{aligned}
$$

```
{ x >= 0 }
while x > 0 do invariant x >= 0
  x := x-1
done
{ x = 0 }
```

wlp: *Computation*

$$\begin{aligned} wlp(\textbf{while}\ldots\textbf{done}, x = 0) = x \geq 0 \land \\ \forall x. x \geq 0 \rightarrow (x > 0 \rightarrow wlp(\textbf{x:=x-1}, x \geq 0)) \land \\ (\neg(x > 0) \rightarrow x = 0) \end{aligned}$$

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{ x >= 0 }
while x > 0 do invariant x >= 0
  x := x-1
done
{ x = 0 }
```

wlp: Computation

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\begin{aligned}
wlp(\textbf{while\ldots done}, x = 0) = x \geq 0 \wedge \\
\forall x. x \geq 0 \to (x > 0 \to x - 1 \geq 0) \wedge \\
(\neg(x > 0) \to x = 0)
\end{aligned}
$$

```
{ x >= 0 }
while x > 0 do invariant x >= 0
  x := x-1
done
{ x = 0 }
```

wlp: *Computation*

This is the verification condition:

$$x \geq 0 \longrightarrow wlp(\textbf{while} \ldots \textbf{done}, x = 0)$$

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wlp: *Computation*

This is the verification condition:

$$
\begin{aligned}
& x \geq 0 \longrightarrow x \geq 0 \wedge \\
& \quad \forall x . x \geq 0 \rightarrow (x > 0 \rightarrow x - 1 \geq 0) \wedge (\neg(x > 0) \rightarrow x = 0)
\end{aligned}
$$

This is what Why3 will do for you:

- Why3 computes (more or less) exactly this.
- Why3 will also do this proof for you.

What do we want to prove?

Partial correctness $\{P\}$ **S** $\{Q\}$

Termination Prove that **S** terminates.

Total correctness Partial correctness + termination

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Partial correctness $\{P\}$ **S** $\{Q\}$

Termination Prove that **S** terminates.

Total correctness Partial correctness + termination

Partial correctness on its own can be weak.

```
while not sorted a do
  tmp := a[0]; a[0] := a[1]; a[1] := tmp;
done
{ sorted a }
```

Proving termination

To prove termination of `while` loops, find some quantity that:

- Gets smaller every iteration
- Never becomes negative

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We call this the **variant**.

```
{ x >= 0 }
while x > 0 do
  invariant x >= 0
  variant x
  x := x-1
done
{ x = 0 }
```

Practical matters: the Why3 toolbox

Programming in WhyML

If you can program, you can program in WhyML!

- Programs can be run directly (`why3 execute`)

Built-in types: `bool`, `int`, `real`

- Data is **immutable** by default
- Mutable data can be stored in *references*: `ref int`

WhyML has annotations for:

- Function contracts: `requires`, `ensures`
- While loops: `invariant`, `variant`
- Assertions: `assert`

WhyML programs

```
let foo (x: int): int
  requires { ... }
  ensures { ... }
= let z:   ref int = ref 0 in
  let odd: ref int = ref 1 in
  let sum: ref int = ref 1 in
  while !sum <= x do
    invariant { ... }
    variant { ... }
    z := !z + 1;
    odd := !odd + 2;
    assert { ... };
    sum := !sum + !odd;
  done;
  return !z
```

Logical language

Pure WhyML expressions + first order logic

Quick syntax guide:

$x \wedge y$	`x/\y`
$x \vee y$	`x\/y`
$\neg y$	`not x`
$x \rightarrow y$	`x->y`
$x \leftrightarrow y$	`x<->y`
$\forall x \in T[P(x)]$	`forall x:t.  p x`
$\exists x \in T[P(x)]$	`exists x:t.  p x`

Logic vs. Code

Pure logic:

```
function double (x: int): int =
  2*x


predicate divides (d n: int) =
  exists q: int. n = q*d
```

Program code:

```
let double (x: int): int =
  2*x


let divides (d n: int): bool =
  d = 0 && n = 0 || mod n d = 0
```

Logic vs. Code

❶ Logical expressions can only be used in annotations

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Logic vs. Code

❶ Logical expressions can only be used in annotations

❷ To reason about programs, you generally need contracts

Pure logic:

```
function double (x: int): int =
  2*x


predicate divides (d n: int) =
  exists q: int. n = q*d
```

Program code:

```
let double (x: int): int
  ensures { result = 2*x }
= 2*x

let divides (d n: int): bool
  ensures { result <->
            exists q: int. n = q*d }
= d = 0 && n = 0 || mod n d = 0
```

Logic vs. Code

Often we can avoid repeating ourselves:

Usable in both logical formulas and programs:

```
let function double (x: int): int =
  2*x

let predicate divides (d n: int)
  ensures { result <-> exists q: int. n = q*d }
= d = 0 && n = 0 || mod n d = 0
```

Why3 IDE

Proving programs is done in the Why3 IDE (`why3 ide`)

- Has a *program view* and a *logical view*
- Allows editing the program
- Access provers with right click
- Logical formulas can be manipulated
  - Strategies: automated splitting & proving
  - Transformations: fine-grained control
- State can be saved and returned to later

Let’s do something!

Example

What does this compute?

```
let foo (x: int): int
= let z   = ref 0 in
  let odd = ref 1 in
  let sum = ref 1 in
  while !sum <= x do
    z := !z + 1;
    odd := !odd + 2;
    sum := !sum + !odd;
  done;
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```

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```

!z	!odd	!sum

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  while !sum <= x do
    z := !z + 1;
    odd := !odd + 2;
    sum := !sum + !odd;
  done;
  return !z
```

!z	!odd	!sum
0	1	1

Example

What does this compute?

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let foo (x: int): int
= let z   = ref 0 in
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  while !sum <= x do
    z := !z + 1;
    odd := !odd + 2;
    sum := !sum + !odd;
  done;
  return !z
```

!z	!odd	!sum
0	1	1
1	3	4

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  let odd = ref 1 in
  let sum = ref 1 in
  while !sum <= x do
    z := !z + 1;
    odd := !odd + 2;
    sum := !sum + !odd;
  done;
  return !z
```

!z	!odd	!sum
0	1	1
1	3	4
2	5	9

Example

What does this compute?

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let foo (x: int): int
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  let sum = ref 1 in
  while !sum <= x do
    z := !z + 1;
    odd := !odd + 2;
    sum := !sum + !odd;
  done;
  return !z
```

!z	!odd	!sum
0	1	1
1	3	4
2	5	9
3	7	16

Example

What does this compute?

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let foo (x: int): int
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  let odd = ref 1 in
  let sum = ref 1 in
  while !sum <= x do
    z := !z + 1;
    odd := !odd + 2;
    sum := !sum + !odd;
  done;
  return !z
```

!z	!odd	!sum
0	1	1
1	3	4
2	5	9
3	7	16
4	9	25

Example

What does this compute?

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let foo (x: int): int
= let z   = ref 0 in
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  while !sum <= x do
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!z	!odd	!sum
0	1	1
1	3	4
2	5	9
3	7	16
4	9	25
5	11	36

Example

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  done;
  return !z
```

!z	!odd	!sum
0	1	1
1	3	4
2	5	9
3	7	16
4	9	25
5	11	36
6	13	49

Example

What does this compute?

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let foo (x: int): int
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  let odd = ref 1 in
  let sum = ref 1 in
  while !sum <= x do
    z := !z + 1;
    odd := !odd + 2;
    sum := !sum + !odd;
  done;
  return !z
```

!z	!odd	!sum
0	1	1
1	3	4
2	5	9
3	7	16
4	9	25
5	11	36
6	13	49
7	15	64

Demo!

How to find good invariants?

[redacted]

Find a property that generalizes initial and end condition.

```
assert { !i = 0 };
let j = ref 9 in
while !i < 10 do
  invariant { ... }
  i := !i + 1;
  j := !j - 1;
done
assert { !i = 9 };
```

How to find good invariants?

Find a property that generalizes initial and end condition.

```
assert { !i = 0 };
let j = ref 9 in
while !i < 10 do
  invariant { !i + !j = 9 }
  i := !i + 1;
  j := !j - 1;
done
assert { !i = 9 };
```

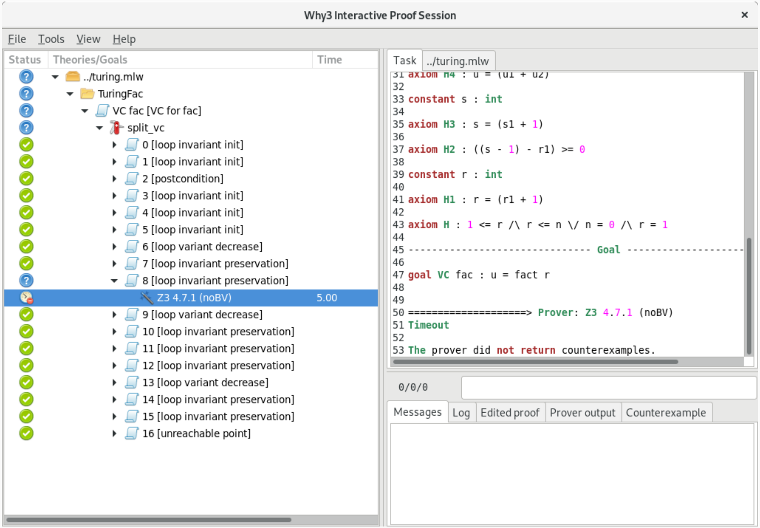
Conclusion

Summary

- Why3 is a platform for automating total correctness proofs
- Uses powerful SMT solvers to do tedious proofs
- WhyML: Logical formulas + Program code
- Functions: specify *contracts*
- `while` loops: specify *invariants* and *variants*

Final comment

Step 1: Frustration



Status	Theories/Goals	Time
?	../turing.mlw	
?	TuringFac	
?	VC fac [VC for fac]	
?	split_vc	
✓	0 [loop invariant init]	
✓	1 [loop invariant init]	
✓	2 [postcondition]	
✓	3 [loop invariant init]	
✓	4 [loop invariant init]	
✓	5 [loop invariant init]	
✓	6 [loop variant decrease]	
✓	7 [loop invariant preservation]	
?	8 [loop invariant preservation]	
	Z3 4.7.1 (noBV)	5.00
✓	9 [loop variant decrease]	
✓	10 [loop invariant preservation]	
✓	11 [loop invariant preservation]	
✓	12 [loop invariant preservation]	
✓	13 [loop variant decrease]	
✓	14 [loop invariant preservation]	
✓	15 [loop invariant preservation]	
✓	16 [unreachable point]	

```
axiom H4 : u = (u1 + u2)

constant s : int

axiom H3 : s = (s1 + 1)

axiom H2 : ((s - 1) - r1) >= 0

constant r : int

axiom H1 : r = (r1 + 1)

axiom H : 1 <= r /\ r <= n \/ n = 0 /\ r = 1

------------------------------- Goal -------------------

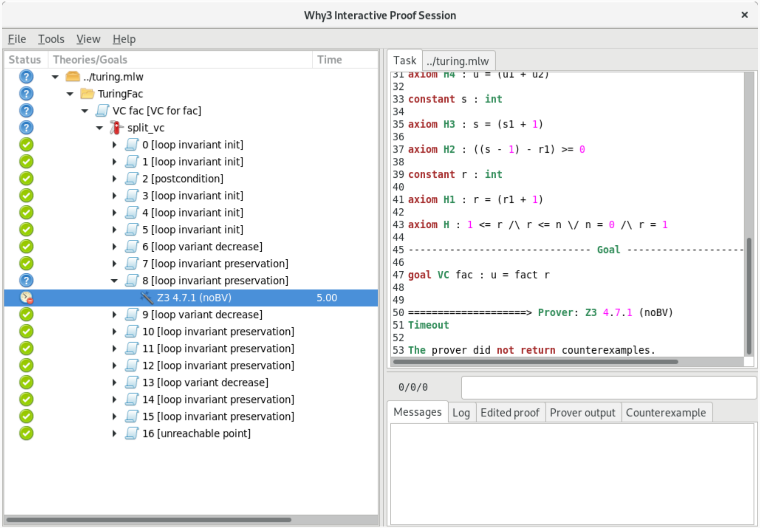
goal VC fac : u = fact r


===================> Prover: Z3 4.7.1 (noBV)
Timeout

The prover did not return counterexamples.
```

Final comment

Step 2: ...



Status	Theories/Goals	Time
?	../turing.mlw	
?	TuringFac	
?	VC fac [VC for fac]	
?	split_vc	
✓	0 [loop invariant init]	
✓	1 [loop invariant init]	
✓	2 [postcondition]	
✓	3 [loop invariant init]	
✓	4 [loop invariant init]	
✓	5 [loop invariant init]	
✓	6 [loop variant decrease]	
✓	7 [loop invariant preservation]	
?	8 [loop invariant preservation]	
⏱	Z3 4.7.1 (noBV)	5.00
✓	9 [loop variant decrease]	
✓	10 [loop invariant preservation]	
✓	11 [loop invariant preservation]	
✓	12 [loop invariant preservation]	
✓	13 [loop variant decrease]	
✓	14 [loop invariant preservation]	
✓	15 [loop invariant preservation]	
✓	16 [unreachable point]	

```
axiom H4 : u = (u1 + u2)

constant s : int

axiom H3 : s = (s1 + 1)

axiom H2 : ((s - 1) - r1) >= 0

constant r : int

axiom H1 : r = (r1 + 1)

axiom H : 1 <= r /\ r <= n \/ n = 0 /\ r = 1

------------------------------- Goal -------------------

goal VC fac : u = fact r

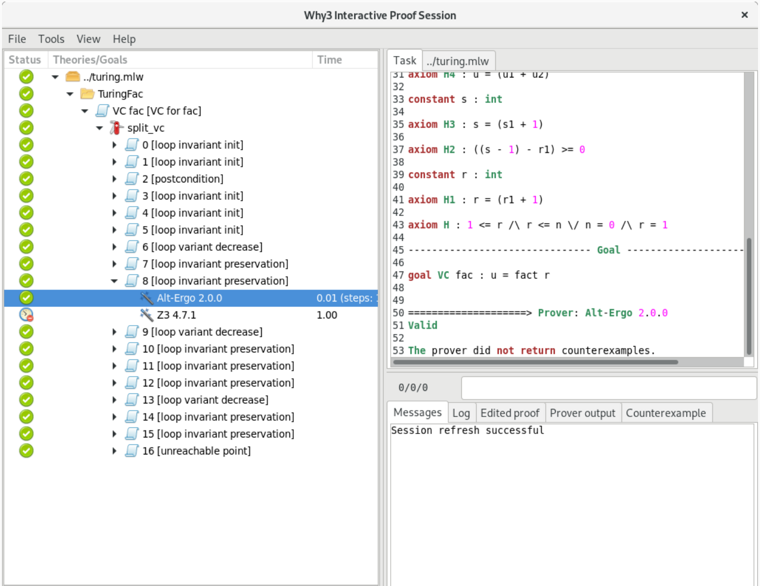

====================> Prover: Z3 4.7.1 (noBV)
Timeout

The prover did not return counterexamples.
```

```
0/0/0
```

Final comment

Step 3: Dopamine rush!



Status	Theories/Goals	Time
✓	../turing.mlw	
✓	TuringFac	
✓	VC fac [VC for fac]	
✓	split_vc	
✓	0 [loop invariant init]	
✓	1 [loop invariant init]	
✓	2 [postcondition]	
✓	3 [loop invariant init]	
✓	4 [loop invariant init]	
✓	5 [loop invariant init]	
✓	6 [loop variant decrease]	
✓	7 [loop invariant preservation]	
✓	8 [loop invariant preservation]	
✓	Alt-Ergo 2.0.0	0.01 (steps:
⏲	Z3 4.7.1	1.00
✓	9 [loop variant decrease]	
✓	10 [loop invariant preservation]	
✓	11 [loop invariant preservation]	
✓	12 [loop invariant preservation]	
✓	13 [loop variant decrease]	
✓	14 [loop invariant preservation]	
✓	15 [loop invariant preservation]	
✓	16 [unreachable point]	

```
axiom H4 : u = (u1 + u2)

constant s : int

axiom H3 : s = (s1 + 1)

axiom H2 : ((s - 1) - r1) >= 0

constant r : int

axiom H1 : r = (r1 + 1)

axiom H : 1 <= r /\ r <= n \/ n = 0 /\ r = 1

------------------------------- Goal -------------------

goal VC fac : u = fact r


===================> Prover: Alt-Ergo 2.0.0
Valid

The prover did not return counterexamples.
```

```
0/0/0
```

```
Session refresh successful
```