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# Algebraic and Higher-Order Differential Cryptanalysis of Pyjamask-96

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Pyjamask is a $2^{\text{nd}}$-round candidate for the NIST lightweight competition

By Goudarzi, Jean, Kölbl, Peyrin, Rivain, Sasaki and Sim.

- Pyjamask-128-AEAD
  - based on Pyjamask-128
  - uses OCB as mode
- Pyjamask-96-AEAD
  - based on Pyjamask-96
  - uses OCB as mode

We focused on the block cipher Pyjamask-96.

Key recovery attack on full-round Pyjamask-96

# Round function

Pyjamask-96 state ($x_i \in \{0,1\}$):

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{16}$ | $x_{17}$ | $x_{18}$ | $x_{19}$ | $x_{20}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ | $x_{25}$ | $x_{26}$ | $x_{27}$ | $x_{28}$ | $x_{29}$ | $x_{30}$ | $x_{31}$ |
| $x_{32}$ | $x_{33}$ | $x_{34}$ | $x_{35}$ | $x_{36}$ | $x_{37}$ | $x_{38}$ | $x_{39}$ | $x_{40}$ | $x_{41}$ | $x_{42}$ | $x_{43}$ | $x_{44}$ | $x_{45}$ | $x_{46}$ | $x_{47}$ | $x_{48}$ | $x_{49}$ | $x_{50}$ | $x_{51}$ | $x_{52}$ | $x_{53}$ | $x_{54}$ | $x_{55}$ | $x_{56}$ | $x_{57}$ | $x_{58}$ | $x_{59}$ | $x_{60}$ | $x_{61}$ | $x_{62}$ | $x_{63}$ |
| $x_{64}$ | $x_{65}$ | $x_{66}$ | $x_{67}$ | $x_{68}$ | $x_{69}$ | $x_{70}$ | $x_{71}$ | $x_{72}$ | $x_{73}$ | $x_{74}$ | $x_{75}$ | $x_{76}$ | $x_{77}$ | $x_{78}$ | $x_{79}$ | $x_{80}$ | $x_{81}$ | $x_{82}$ | $x_{83}$ | $x_{84}$ | $x_{85}$ | $x_{86}$ | $x_{87}$ | $x_{88}$ | $x_{89}$ | $x_{90}$ | $x_{91}$ | $x_{92}$ | $x_{93}$ | $x_{94}$ | $x_{95}$ |

- `AddRoundKey`: linear key schedule applied to key of 128 bits
- `SubBytes`: a 3-bit S-box of degree 2
- `MixRows`: circulant binary matrix to rows

Pyjamask-96 consists of 14 rounds.

**Definition (Derivative [Lai, 1994])**

Let $F\colon \mathbb{F}_2^n \to \mathbb{F}_2^n$ and $a \in \mathbb{F}_2^n$ be given.

Then the derivative of $F$ to $a$, $\Delta_a F$ is: $\Delta_a F(x) = F(x+a) + F(x)$.

Properties:

- $\Delta_{a_k}\Delta_{a_{k-1}}\cdots\Delta_{a_1}F(x) = \sum_{v\in[\![ a_1,\ldots,a_k ]\!]} F(x+v) =: \Delta_V F(x)$
- $\deg \Delta_V F(x) \leq \deg F - \dim V$
- If $\dim V > \deg F$, then we have $\Delta_V F(x) = 0$

Degrees of the $n$-round versions of Pyjamask-96 are upper bounded by

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11+ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| degree | 2 | 4 | 8 | 16 | 32 | 64 | 80 | 88 | 92 | 94 | 95 |

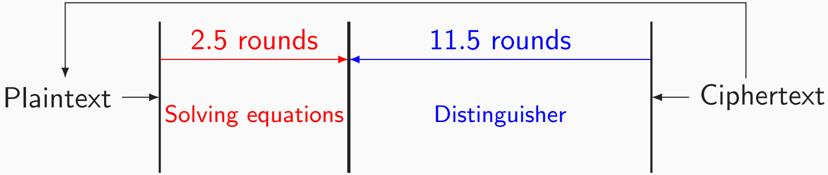
Bounds by Boura, Canteaut, De Cannière [2011]

Affine spaces $V$ of dimension 94 give distinguisher

$$\sum_{v \in V} \mathrm{Pyj}_K^{10}(x+v) = C^{\mathrm{st}}$$

Same for the inverse of Pyjamask-96!

# Meet-in-the-middle




- Smartly choosing affine ciphertext space gives 11 rounds instead
- $\mathcal{U} = \{u \in \mathbb{F}_2^{96} \mid u_0 = u_{32} = u_{64} = 0\}$ has codimension 3
- $V_0 = \{0, v\}$ where $v_i = 0$ for all $i \in \{1, \ldots, 31, 33, \ldots, 63, \ldots, 95\}$
- $\mathcal{V} = \mathcal{U} \oplus V_0$ has dimension 94 and
- $\sum_{v \in \mathcal{V}} \mathrm{Pyj}_K^{11}(x + v)$ constant

# Solving equations

- Taking key-bits as variables gives system of equations
- Linearise to solve linear system of monomials
  - Full codebook gives 448 equations
  - Too many monomials

# Reducing monomials

- Reducing in S-box:

$$\begin{aligned} S(P+K)_0 &= (p_0+k_0)(p_2+k_2)+p_1+k_1 \\ &= S(P)_0+S(K)_0+p_0k_2+p_2k_0 \end{aligned}$$

- Applying further `MixRows` and `AddRoundKey`:

$$(L\circ S)(P)+(L\circ S)(K_0)+K_1+\sum_{\substack{i,j\in I \\ |I|=11,13}} p_ik_j+p_jk_i$$

  - Equivalent key: $\kappa=(L\circ S)(K_0)+K_1$,
  - Equivalent plaintext: $P'=(L\circ S)(P)$
- Still too many monomials

# Guess-and-determine

- Guess-and-determine on roundkey bits
  - Guess all bits in first roundkey:
  - 96 guesses $\rightarrow$ 569 monomials
  - Guess four more bits in the second roundkey:
  - 100 guesses $\rightarrow$ 411 monomials
- Introduces a $2^{100}$ factor in computation

# Complexities

| Rounds | Time (in Pyjamask-96 calls) | Data (in blocks) |
|---|---|---|
| 14/14 | $2^{115}$ | $2^{96}$ |
| 13/14 | $2^{99}$ | $2^{96}$ |
| 12/14 | $2^{96}$ | $2^{96}$ |
| 11/14 | $2^{91}$ | $2^{95}$ |
| 10/14 | $2^{83}$ | $2^{87}$ |
| 9/14 | $2^{67}$ | $2^{71}$ |
| 8/14 | $2^{35}$ | $2^{39}$ |
| 7/14 | $2^{27}$ | $2^{23}$ |

# Further research

- Attacking Pyjamask-96-AEAD
  - We got to 7 rounds with $2^{86}$ time complexity, $2^{41}$ data.
- Attacking Pyjamask-128-AEAD