## Is $\chi_n$ a power function?

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## Introduction

Introduction	
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Proof technique

## Introduction to $\chi_n$

Definition 1 ( $\chi_n$ )

The map  $\chi_n \colon \mathbb{F}_2^n \to \mathbb{F}_2^n, \ x \mapsto y$  is given by:

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ASCON, the Lightweight Cryptography winner:  $\chi_5$ .

Proof technique

## Properties of $\chi_n$

•  $\chi_n$  has (algebraic) degree 2:  $y_i = x_i + x_{i+1}x_{i+2} + x_{i+2}$ ;

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- $\operatorname{ord}(\chi_n) = 2^{\lfloor \lg n \rfloor}.$

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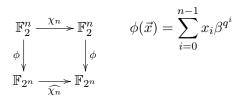
### Univariate polynomials



$$\phi(\vec{x}) = \sum_{i=0}^{n-1} x_i \beta^{q^i}$$

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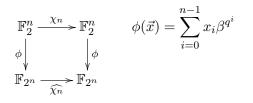


#### Definition 2 (Normal basis)

Consider  $\mathbb{F}_2 \subset \mathbb{F}_{2^n}$ . Then  $\beta \in \mathbb{F}_{2^n}$  is called a *normal element* of  $\mathbb{F}_{2^n}$  if the set  $\{\beta, \beta^2, \beta^{2^2}, \dots, \beta^{2^{n-1}}\}$  is a linear independent set. This set is then called a *normal basis* of  $\mathbb{F}_{2^n}$ .

Proof technique

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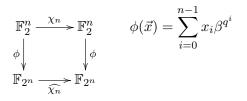
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#### Theorem 3

If  $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$  is shift invariant and the isomorphism  $\phi$  is induced by a normal element, then  $\widehat{F}$  has coefficients in  $\mathbb{F}_2$ .

Proof technique 00000

### Univariate polynomials



#### Example 2

Consider the map  $\chi_3$ . Let  $\mathbb{F}_{2^3} := \mathbb{F}_2(\alpha) = \mathbb{F}_2[X]/(X^3 + X + 1)$ . Then  $\alpha^3$  is a normal element. We define  $\widehat{\chi_3} := \phi \circ \chi_3 \circ \phi^{-1}$ . By using Lagrange interpolation we find that  $\widehat{\chi_3}(t) = t^6$ .

### Power functions

#### Definition 3 (Power functions)

A power function is a polynomial function that can be represented by a single monomial in  $\mathbb{F}_{2^n}[X]$ . We write  $*^e \colon \mathbb{F}_{2^n} \to \mathbb{F}_{2^n}$  for a power function, here  $e \ge 0$ .

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Why power functions?

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#### Proposition 1

For any even n, there is no (normal) basis representation such that  $\widehat{\chi_n}$  is a power function.

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#### Proof.

Suppose that it does exist. Since  $\chi_n((01)^{n/2}) = 0^n$ , there needs to exist some nonzero  $\alpha \in \mathbb{F}_{2^n}$  with  $\alpha^s = 0$  for some integer s.  $\Box$ 

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Proposition 1 (Excluding Mersenne-exponents)

If n > 3 is such that  $2^n - 1$  is a prime number, then there exists no (normal) basis representation of  $\chi_n$  such that  $\widehat{\chi_n}$  is a power function.

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#### Proof.

Since  $2^n - 1$  is a prime number,  $\varphi(2^n - 1) = 2^n - 2$ . The order of  $\chi_n$  is divisible by 4 for all n > 3. The expression  $2^n - 2$  has only one factor 2.

### State diagrams

#### Definition 4 (State diagram)

Let S be a set. The state diagram for a map  $F: S \to S$  is a directed graph (V, A), where V = S and  $A = \{(a, F(a)) \mid a \in S\}$ .

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#### Theorem 5 (Ahmad's Theorem)

Let m, q be positive integers with  $q = p^n$  for some prime number pand  $n \ge 1$ . Let  $*^e \colon \mathbb{F}_q^* \to \mathbb{F}_q^*$ ,  $x \mapsto x^e$  be a power function. Then  $*^e$  has a cycle of length precisely m if and only if there exists some  $t \mid q-1$  such that the order of e modulo t is equal to m.

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Not every length necessarily occurs!

# Corollary

Theorem 6 (Necessary conditions for  $\chi_n$  to be a power function)

Let n > 3 be an odd integer. Write  $o := \operatorname{ord}(\chi_n) = 2^{\lfloor \lg(n) \rfloor}$ . Then  $\chi_n$  can only be a power function if  $2^n - 1$  factors as

$$2^n - 1 = p_1^{e_1} \cdots p_r^{e_r},$$

such that there exists some permutation  $\sigma \in S_r$  with

$$\begin{split} \varphi(p_{\sigma(1)}^{e_{\sigma(1)}}) & \text{is a multiple of } o \\ \varphi(p_{\sigma(2)}^{e_{\sigma(2)}}) & \text{is a multiple of } \frac{o}{2} \\ & \vdots \\ \varphi(p_{\sigma(t)}^{e_{\sigma(t)}}) & \text{is a multiple of } 2 \end{split}$$

for some t < r.

Introduction	Historical attempts	Proof technique
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Results		

Using these conditions, we can verify<sup>1</sup> that  $\chi_n$  is not a power function for any  $n \leq 1115$ , except for n = 63 and n = 441.

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Remaining cases:

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•  $\approx 2^{12.59}$  out of  $\approx 2^{62.742}$  possible e;

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## Proof technique

### Differential distributions

Definition 7 (Differential probability (Biham, Shamir))

Let  $f: G \to H$  be a map between finite groups G and H. Let  $g \in G$  and  $h \in H$  be arbitrary. Then we define the differential probability of f at (g, h) as

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		input difference								
	$\chi_3$	000	001	010	011	100	101	110	111	
output difference	000	1	-	-	-	-	-	-	-	
	001	-	$^{1/4}$	-	$^{1/4}$	-	$^{1/4}$	-	1/4	
	010	-	-	$^{1/4}$	1/4	-	-	1/4	1/4	
	011	-	$^{1/4}$	1/4	-	-	$^{1/4}$	1/4	_	
	100	-	-	-	-	1/4	1/4	1/4	1/4	
	101	-	$^{1/4}$	-	$^{1/4}$	1/4	-	1/4	_	
no	110	-	-	$^{1/4}$	1/4	1/4	$^{1/4}$	-	-	
	111	-	1/4	1/4	-	1/4	-	-	1/4	

Proof technique

# Differential distribution for $\chi$

Proposition 2 (Differential probabilities for  $\chi$  (Daemen))

Let n > 1 be an arbitrary odd integer. Let  $a \in \mathbb{F}_2^n$  be arbitrary. Then for any compatible  $b \in \mathbb{F}_2^n$  we have  $\mathrm{DP}_{\chi_n}(a,b) = 2^{-w(a)}$ , where

$$w(a) = egin{cases} n-1 & \text{if } a = 1^n, \ \operatorname{wt}(a) + r & ext{else}, \end{cases}$$

where r is the number of (cyclic) 001-substrings in a.

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Introduction	

### Invariant

Proposition 3 (Differential probabilities under linear isomorphisms)

Let  $G \stackrel{\varphi}{\cong} H$  be isomorphic groups. Let  $f: G \to G$  be a map and let  $\hat{f}: H \to H$  be the map induced through the isomorphism. Then  $\mathrm{DP}_{\hat{f}}(g,h) = \mathrm{DP}_{f}(\varphi^{-1}(g), \varphi^{-1}(h))$  for all  $g, h \in H$ .

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Let  $0 \le e \le 2^n - 1$  and let  $f = *^e \colon \mathbb{F}_{2^n} \to \mathbb{F}_{2^n}$  be a power function. Then  $DP_f(a, b) = DP_f(ya, y^e b)$  for all  $y \in \mathbb{F}_{2^n}^*$ .

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Substitute  $x := yy^{-1}x = yx'$  in  $DP_f(ya, y^eb) = \#\{x \in \mathbb{F}_{2^n} \mid x^e + (x + ya)^e = y^eb\}/2^n.$ 

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Thus, we have that the rows of the DDT all have the same number of occurrences of  $0, 2, 4, \ldots$ 

## Conclusion and corollary

#### Theorem 8

Let  $n \neq 1, 3$  be a positive integer. Then  $\widehat{\chi_n}$  is not a power function.

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There is no function  $F_n$  that is extended affine equivalent to  $\chi_n$  $(AF_nB + C = \chi_n)$ , such that  $\widehat{F_n}$  is a power function.

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Thank you for your attention!